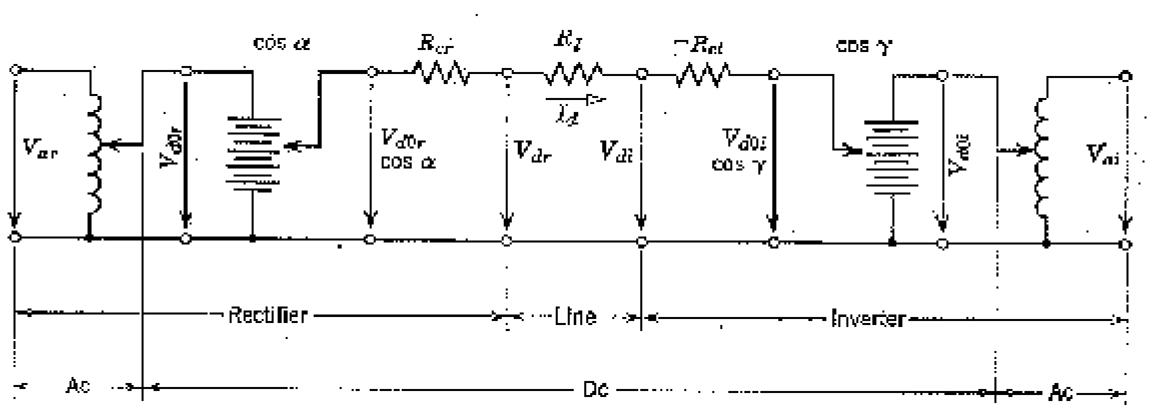
UNIT-III

HVDC CONTROL

Basic means of control:

OVERALL EQUIVALENT CIRCUIT OF HVDC SYSTEM



From the overall equivalent circuit of HVDC system

$$I_d = \frac{V_d \; c \; \alpha - V_d \; c \; (\beta/\gamma)}{R_C + R_L \pm R_C}$$

The DC voltage and current in the DC link can be controlled by controlling rectifier voltages and inverter voltages using two methods

- **GRID CONTROL**
- **MANUAL CONTROL**

- **GRID CONTROL:** It is done by varying ignition angle of the valves. It is rapid or instantaneous control

- **MANUAL CONTROL:** It is done changing the taps ratio of the converter transformer. It is slow and done in steps
  Power reversal can be done by changing the polarity of the DC voltage at both ends

BASIS FOR SELECTION OF THE CONTROL:

- Prevention of large fluctuating current due to variations of AC voltages
- Maintaining the DC voltage near to its rated
- Maintaining the power factor at the sending and receiving end as high as possible
- Prevention of various faults in the valves

What is the Need for power factor high?

- To keep the rated power in the converter as high as possible wrt given voltage, current, voltage ratings of the transformer and the valves.
- To reduce the stress on the valve.
- To minimize the losses and the current ratings of the equipment in the AC system to which the converter is connected.
- To minimize the voltage drops as the load increases.
- To minimize the reactive power supplied to the converter

DESIRED FEATURES OF THE CONTROLLER:

- Control system should not be sensitive to normal variations in voltage and frequency of the AC supply system.
- Control should be fast reliable and easy to implement.
- There should be continuous operating range of full Rectification to full Inversion.
- Control should be such that it should require less reactive power.
- Under at steady state conditions the valves should be fired symmetrically.
- Control should be such that it must control the maximum current in the DC link and limit the fluctuations of the current.
- Power should be controlled independently and smoothly which can be done by controlling the current or voltage or both.
- Control should be such that it can be used for protection of the line and the converter

CONSTANT VOLTAGE	CONSTANT CURRENT
Voltage is constant	Current is constant
Current is varied to change power	Voltage is varied to change power
Loads and power sources are connected in parallel in order to turnoff a load or a source respective branch is opened	Loads and power sources are connected in series in order to turn off a load or source it should be bypassed
AC transmission and DC Distribution	Street lighting in DC
DC system the fault current can be greater limited by circuit resistance	Short circuit current is ideally limited by load current and it is twice of the rated current and Accidental open circuits give rise to huge voltages
Power loss is α (power transmitted)^2	Power loss is α full load value

From the overall equivalent circuit of HVDC system

$$I_d = \frac{V_d \cos\alpha - V_d \cos(\beta/\gamma)}{R_C + R_L \pm R_C}$$

$$V_d = \frac{V_d}{2}[\cos(\mu+\alpha) + \cos(\alpha)]$$

We know that

$$\cos(\alpha) = 0.5[\cos(\mu+\alpha) + \cos(\alpha)]$$

$$\cos(\alpha) = 0.5[\cos(\mu+\alpha) + \cos(\gamma)]$$

Therefore for achieving high power factor α for rectifier and γ for inverter should be kept as low as possible

The rectifier has minimum α limit of about 5^0 to ensure adequate voltage across the valves before firing . consequently the rectifier normally operates within in the range of 15^0 and 20^0 so as to leave a room for increasing rectifier voltage to control DC power flow.

In the case of the inverter it is necessary to maintain the a certain minimum Extinction angle to avoid commutation failure. It is important to ensure that commutation is completed with sufficient margin to allow deionization before voltage reverses $\gamma = \beta - \mu$. The minimum margin for this is 15^0 to 50Hz and 18^0 for 60Hz supply.

In order to satisfy basic requirements for better voltage regulation and current regulation it is always be advisable to assign these parameters for the converters. Under normal operations **Rectifier** will take care of the **current** and the **Inverter** will take care of the **voltage**.

Rectifier - Constant Current Control (CC)

Inverter - Constant Extinction Angle Control (CEA)

Let us examine how AC voltages changes reflect in the DC current and which controller has to be exercised to make DC link current at rated value.

$$I_d = \frac{V_{dcr}\cos\alpha - V_{dci}\cos(\beta/\gamma)}{R_L + R_L \pm R_L}$$

- **INCREASE IN THE RECTIFIER VOLTAGE:** Current in the DC link will increase to control the current in the rectifier end, controller will increase delay angle α while at the inverter end controller will maintain CEA. Increase in the delay angle worsens the power factor. Generally it is controlled in steps thereafter tap change is done

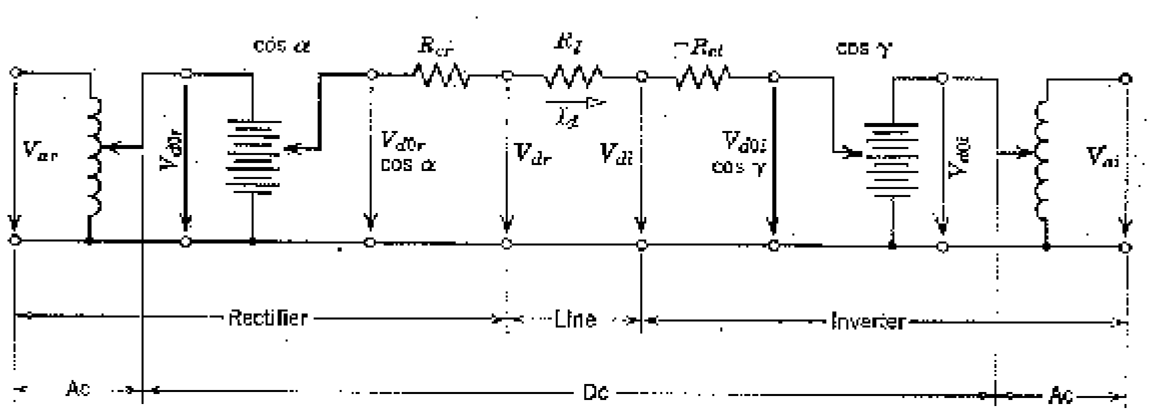
- **INCREASE IN THE INVERTER VOLTAGE** : Current in the DC link will decrease to control the current in the rectifier end, controller will decrease delay angle α up to α min while at the inverter end controller will maintain CEA. decrease in the delay angle improves the power factor. Generally it is controlled in steps thereafter tap change is done

- **DECREASE IN THE RECTIFIER VOLTAGE: :** Current in the DC link will decrease to control the current in the rectifier end, controller will decrease delay angle α up to α min while at the inverter end controller will maintain CEA. decrease in the delay angle improves the power factor. Generally it is controlled in steps thereafter tap change is done. If the further decrease in the rectifier voltage characteristics falls below and CEA characteristics does not intersect then Dc link current will be zero. Therefore inverter also should be equipped with constant current controller

- **DECREASE IN THE INVERTER VOLTAGE**: Current in the DC link will increase to control the current in the rectifier end, controller will increase delay angle α while at the inverter end controller will maintain CEA. Increase in the delay angle worsens the power factor. Generally it is controlled in steps thereafter tap change is done

CHARACTERISTICS OF THE HVDC SYSTEM:

Slope of α, β and γ characteristics



$$V_d = V_d \ \ c \ \ \alpha - R_C I_d$$

$$V_d = V_d \ \ c \ \ (\beta/\gamma) \pm R_C I_d$$

In this setup we have modeled the rectifier and inverter only missing is the line the resistance of the line we have to include the resistance of the line either with rectifier or with the inverter

Voltage drop across the line = $\mathbf{R_L I_d}$

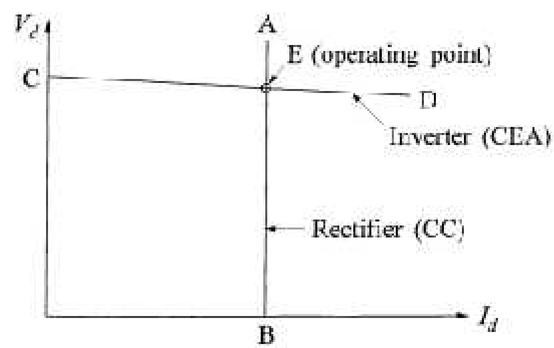
Including the drop along the rectifier we get

$$\mathbf{V_d = V_d \quad c \quad \alpha - (R_C + R_L) I_d}$$

$$\mathbf{V_d = V_d \quad c \quad (\beta) + R_C I_d}$$

$$\mathbf{V_d = V_d \quad c \quad \gamma - R_C I_d}$$

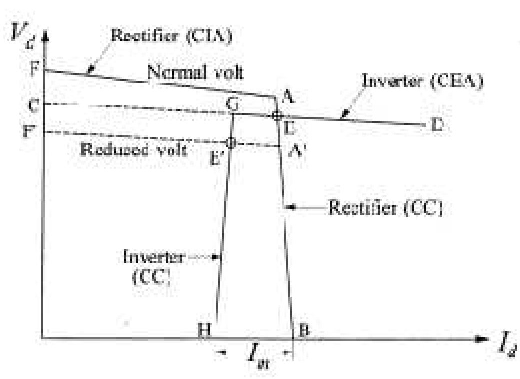
IDEAL CHARACTERISTICS: Rectifier will take care of current so it is a line parallel to Y axis. As Inverter equation with gama is negative slope. The point where rectifier current control and inverter voltage control coincide there exist a operating point which is the power order of the HVDC link



The rectifier characteristics can be shifted horizontally by adjusting the current command or current order. If the measured current is less than the command the regulator advances the firing by decreasing α.

The inverter characteristics can be raised or lowered by means of the transformer tap changer. When the tap is moved the CEA regulator quickly restores the desired gama. As a result the DC current changes which is then quickly restored by current regulator of the rectifier.

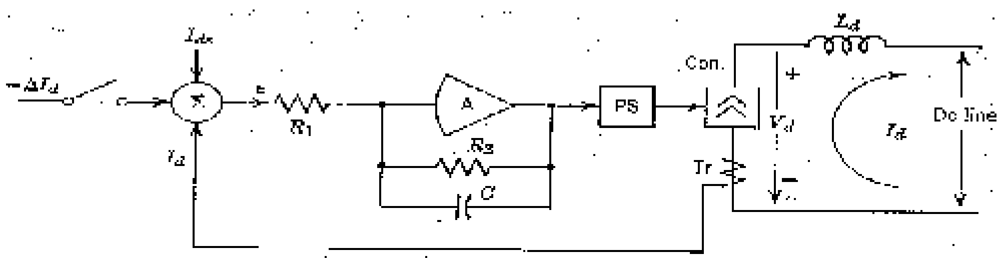
ACTUAL CHARACTERISTICS:

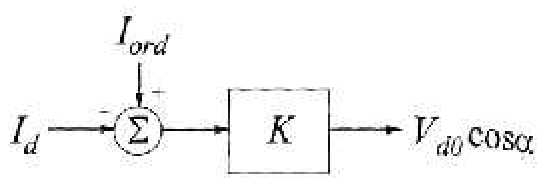


- The rectifier maintains constant current in the DC link by changing α however α cannot be less than α_{min}. Once α_{min}. is hit no further increase in voltage is possible. This is called Constant Ignition Angle Control(CIA)

- In practice as current controller will have a proportional controller it has high negative slope due to finite gain of the controller

CONSTANT CURRENT CONTROLLER:





I_{ord} = current order

With the current regulator gain K

$$V_d \; c \; \alpha = K(I_o - I_d)$$

$$V_d \; c \; \alpha = V_d + R_C I_d$$

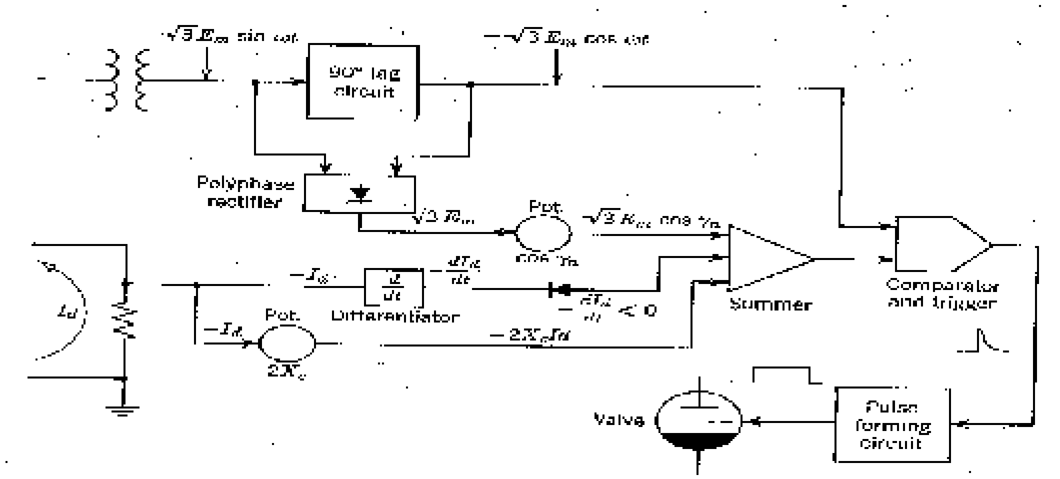
There fore

$$V_d = KI_o - (K + R_C)I_d)$$

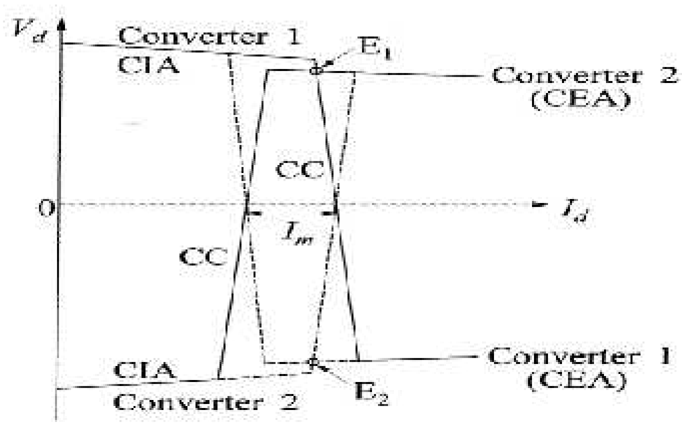
CONSTANT CURRENT CONTROL INVOLVES THE FOLLOWING:

- Measurement of the DC current.
- Comparison of I_d with the set value I_{ds} or I_{ord} (called as Reference/Current Order /Current Command).
- Amplification to the differences called error.
- Application of the output signal of the amplifier to the phase shift circuit that alters the ignition angle α of the valves in the proper direction for reducing the error.

CONSTANT EXTINCTION ANGLE CONTROL:



COMBINED RECTIFIER AND INVERTER CHARACTERISTICS:

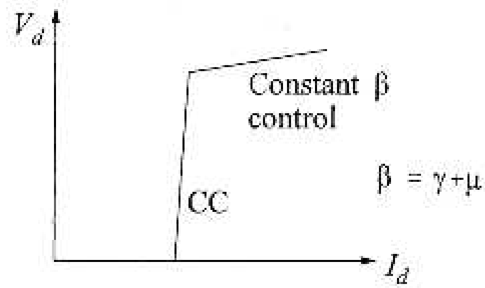


Power reversal can be done by changing the current settings of the converter and inverter which is shown in the dotted line above

ALTERNATIVE CONTROL STRATEGIES FOR INVERTER:

DC VOLTAGE CONTROL MODE: instead of regulating by fixing γ a closed loop may be used so as to maintain the constant voltage at desired point on the DC line. It ensures that the voltage is constant

CONSTANT β CONTROL: In the inverter equation if there is beta there will be a positive slope. At low load beta gives additional security against commutation failure. However for heavy currents and large overlap gama is used



For stabilization and ambiguity reasons also

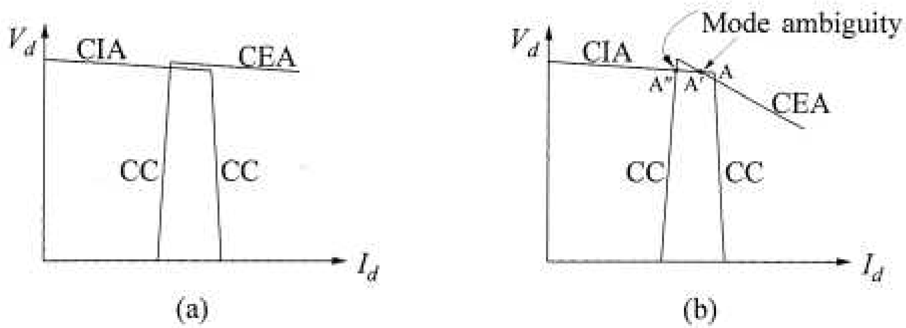
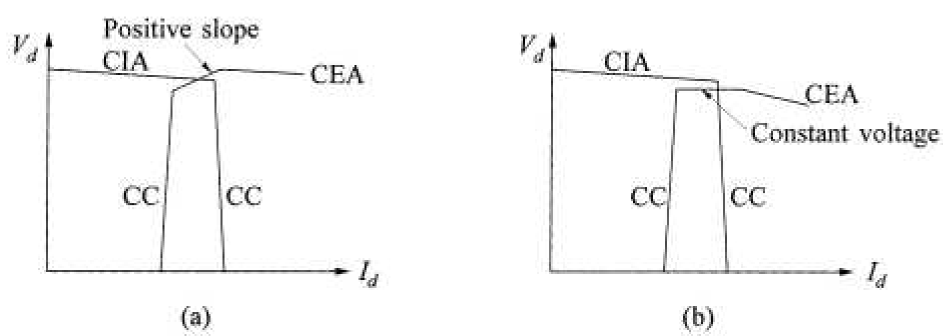


Figure 10.33 Regions of mode ambiguity



TAP CHANGE CONTROL: Tap changing control is done to maintain the firing angles in the desired range. Normally rectifier which takes care of the current backed up by tap change and also inverter CEA also backed up by tap change.

They are changed in fixed steps from minimum to maximum. Tap changes are prevented during transients. Hunting is avoided by having dead band wider than step size.

CURRENT LIMITS

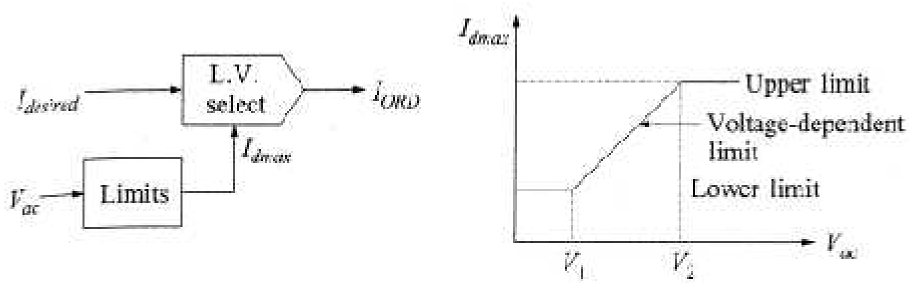
- **MAXIMUM CURRENT LIMITS:** The maximum short time current is usually limited to 1.2 to 1.3 times the normal full load current to avoid thermal damage to the valves.

- **MINIMUM CURRENT LIMIT**: As the load current is discontinuous high voltages may occur in the transformer windings and this can be avoided by high Dc reactor on the DC side

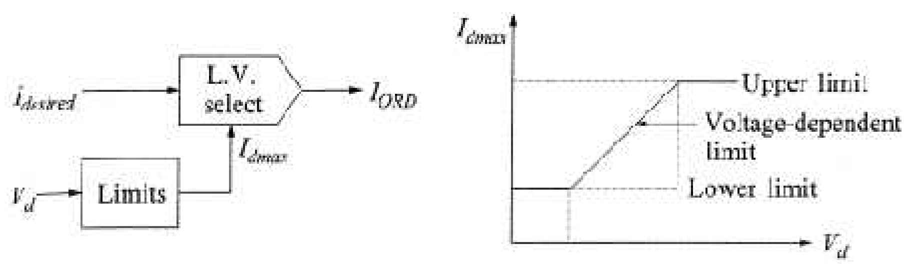
- **VOLTAGE DEPENDENT CURRENT ORDER LIMIT(VDCOL):** under low voltage condition it may not be desirable to maintain DC current or power for the following reasons

  When voltage at one converter falls more than 30%, the reactive power demand of the remote converter increase and this may have adverse effects on the AC system. A higher alpha or Gama is necessary to control the current in the link which increase the reactive power demand at the converter. If the Ac system voltage is reduced substantially the amount of reactive power is reduced

  At reduced voltages there is also chance of commutation failure and voltage instability

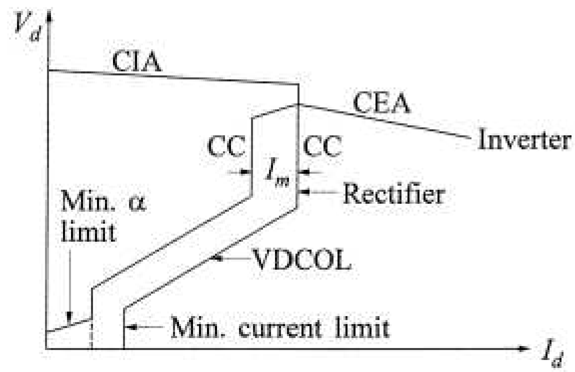


(a) Current limit as a function of alternating voltage



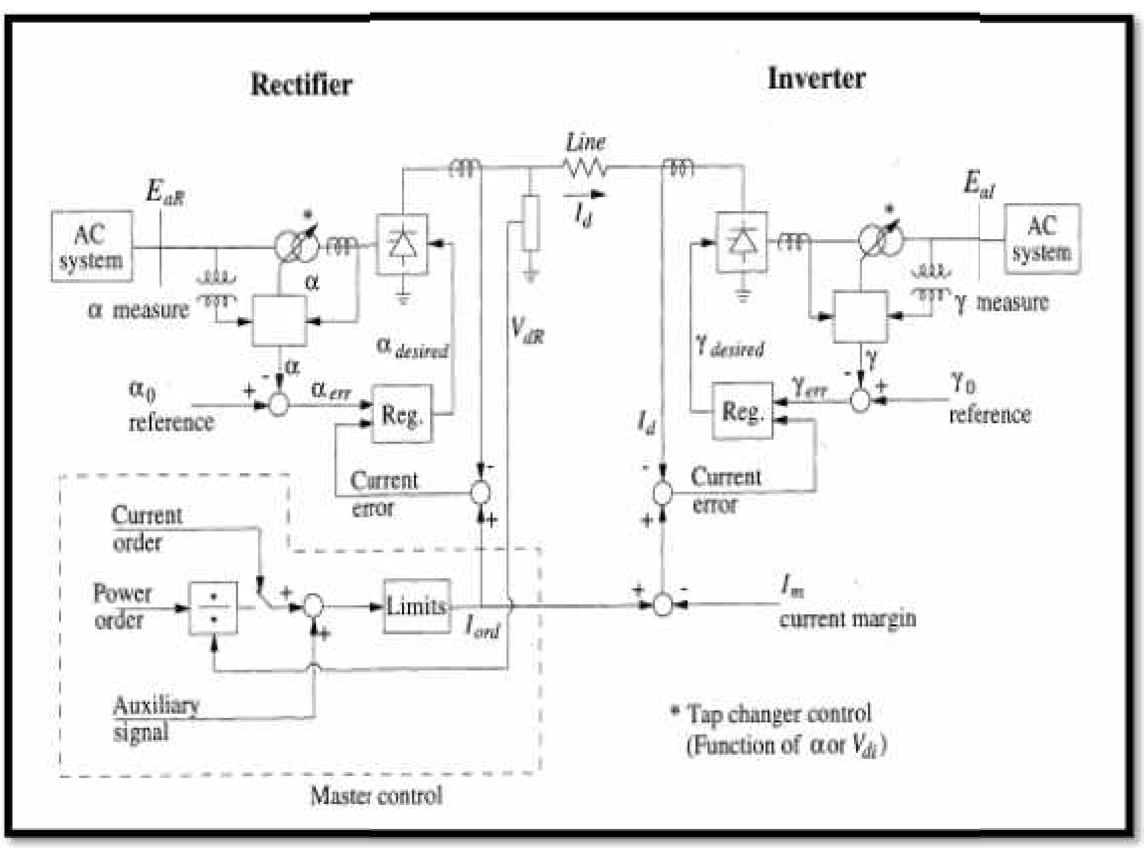
(b) Current limit as a function of direct voltage

VDCOL CONTROL



MINIMUM FIRING ANGLE CONTROL: power transferred in the DC line is mainly due to manipulation of the current order. These signals are to be sent to the converter via telecommunication. If this link fails there is a chance that a inverter can change to rectifier which results in power reversal. To prevent that the inverter control is provided with minimum delay angle control

POWER CONTROL: Usually the HVDC link is required to transmit scheduled power. In such an application the corresponding current order is found out from the power order/ DC Voltage measured



CONTROL HIERARCHY:

Generally two bridges with star- star and star delta connected transformers are considered for 12 pulse bridge unit.

The control scheme is divided into four levels

- **BRIDGE OR CONVERTER LEVEL**
- **POLE CONTROL**
- **MASTER CONTROL**
- **OVERALL CONTROL**

- **BRIDGE OR CONVERTER LEVEL:**

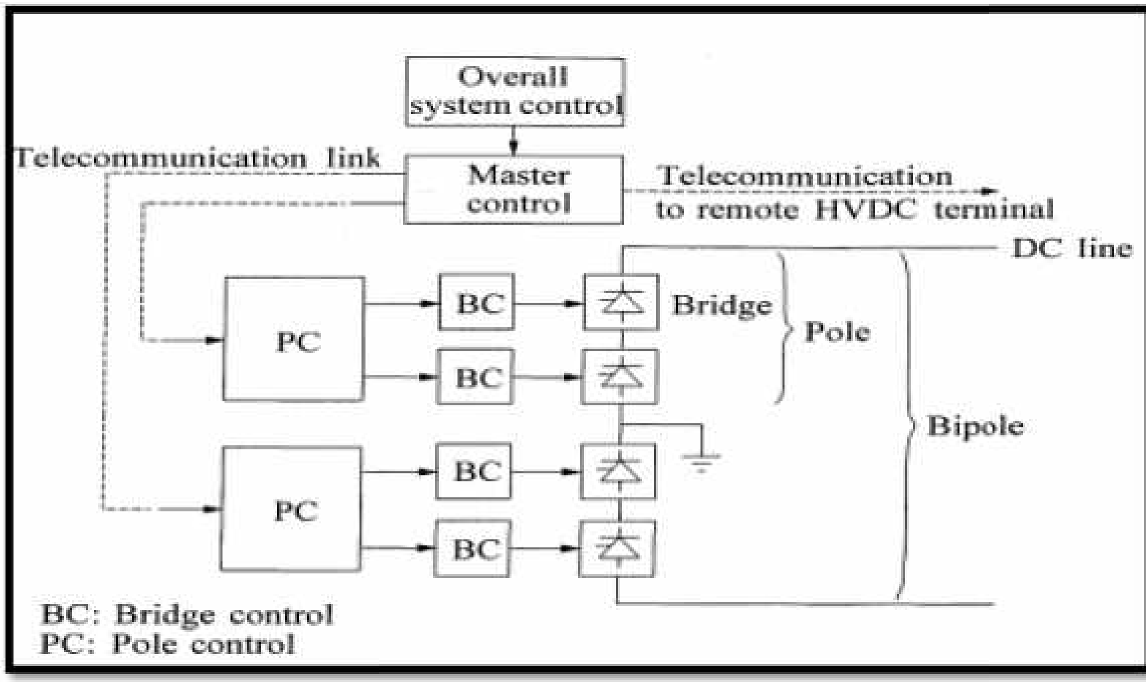
  - It determines the firing instants of the valves within a bridge and defines α_{min} and γ_{min} limits.
  - This has the fast response in the hierarchy.

- **POLE CONTROL:**

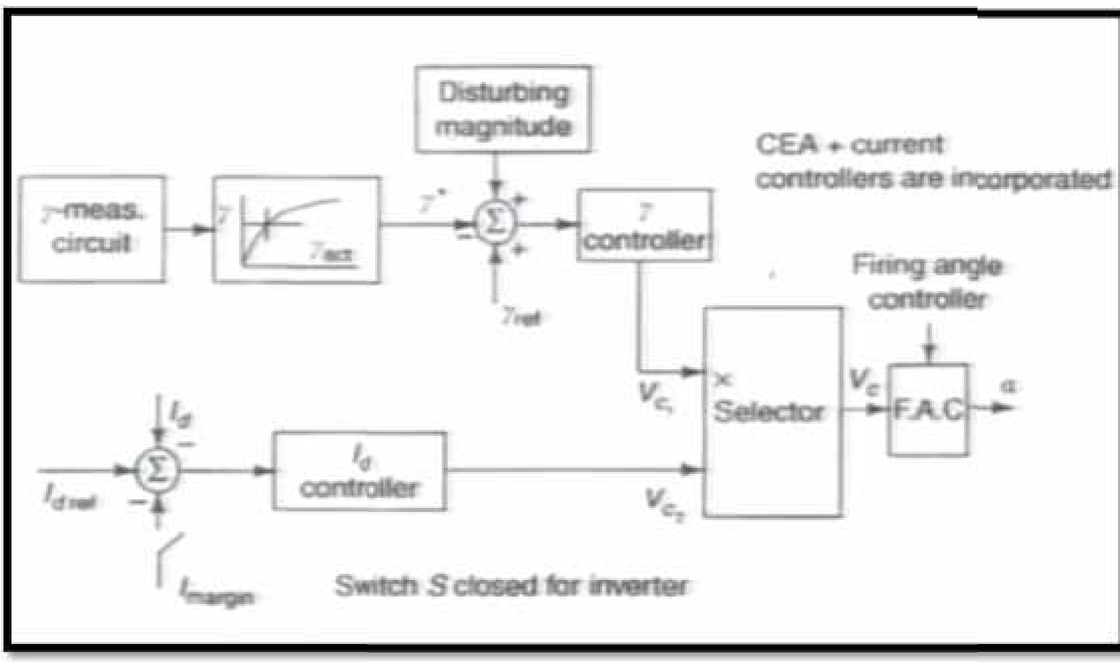
  - It coordinates the control of bridges in a pole.
  - The conversion of the current order to the firing angle order, tap changer control and some protection control sequence are handled in pole control.
  - It also handles starting, stopping and de-blocking and balancing of the bridge

- **MASTER CONTROL:**

  - It determines the current order and provides coordinated current order signals to all poles.
  - It interprets the broader demands for controlling the HVDC system by providing the interface between the pole control and overall system control
  - This includes power flow scheduling determined by control centre and AC system stabilization



BRIDGE/VALVE GROUP CONTROL:



FIRING ANGLE CONTROL: The manner which mode you operate the HVDC system either in CIA,CC and CEA only need is how we generate firing pulses.

There are two methods in which firing pulses can be generated

- **INDIVIDUAL PHASE CONTROL (IPC).**
- **EQUIDISTANT PULSE CONTROL (EPC).**

- **INDIVIDUAL PHASE CONTROL (IPC):**
    - Here firing angles are calculated individually for every phase with their commutation voltages.
    - Six phase delay circuits are required
    - Six zero crossing circuits are required

In this IPC there are two methods

- **COSINE CONTROL**
- **LINEAR CONTROL**

➢ **COSINE CONTROL:** There are several versions of this method.

- Pulses are generated at the zero crossing of control voltage V_C–and the line voltage AC.
- The control voltage is nothing but error produced from the current

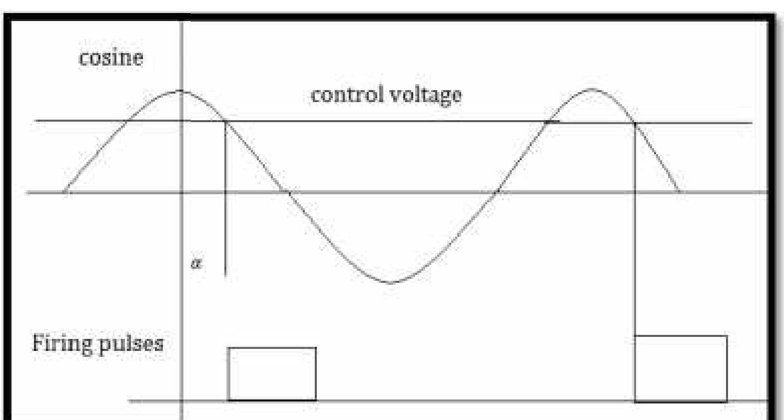
$$V_C = V_m \cos \alpha$$

$$V_d = V_d \cos \alpha$$

$$\alpha = \cos^{-1}\left(\frac{V_C}{V_m}\right)$$

$$V_d = V_d \cos \alpha = KV_C$$

- This control system results in a linear transfer characteristic.
- The output voltage independent on the change of the input AC voltage.
- However near alpha at zero it is sensitive to control voltage and leads to high accuracy

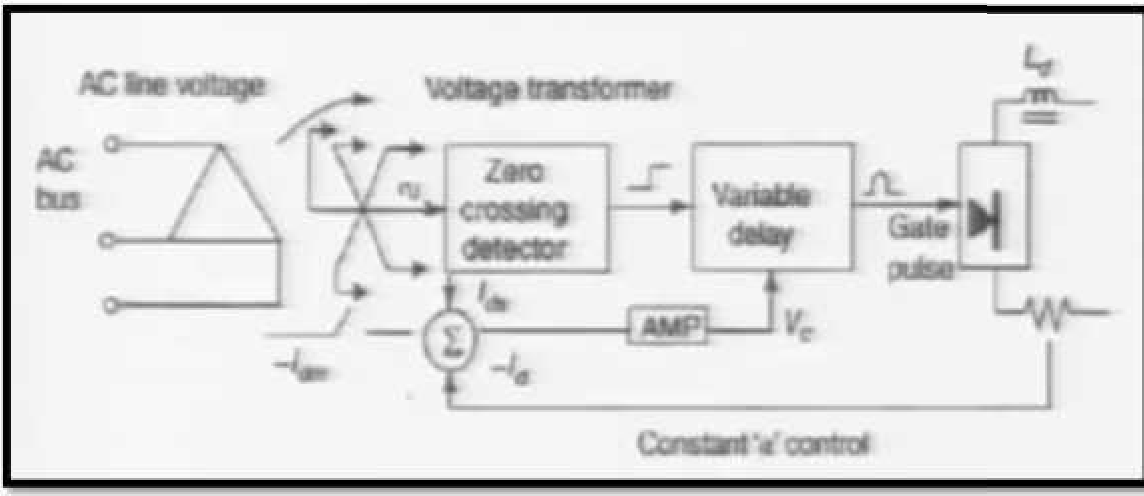


- **LINEAR CONTROL:**
  - Pulses are generated at the zero crossing of control voltage V_C and the line voltage AC.

$$\alpha = K_1V_C$$

$$V_d = V_d \ c \quad \alpha = K_1V_C$$

  - This makes linear transfer characteristics non linear but accuracy is $\pm 1^0$



ADVANTAGES OF IPC:

- The output voltage will be high

DISADVANTAGES OF IPC:

- Harmonic instability with less SCR.
- Non characteristics harmonics introduction in the system.
- Parallel resonance with filter impedance and system impedance

- **EQUIDISTANT PULSE CONTROL (EPC):** Here pulses are generated in the steady state at a equal intervals of 1/Pf. Through the ring counter

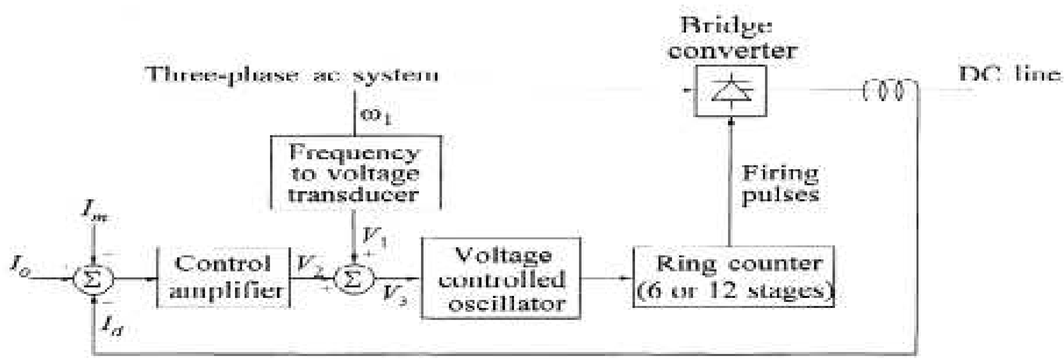
It consists of three variations of the EPC scheme

- **PULSE FREQUENCY CONTROL (PFC)**
- **PULSE PEROID CONTROL (PPC)**
- **PULSE PHASE CONTROL (PPC)**

❖ **PULSE FREQUENCY CONTROL (PFC):**

- The basic components of the system are Voltage Controlled Oscillator (VCO) and a ring counter. The VCO delivers pulses at a frequency directly proportional to the input control voltage.

- The train of pulses is fed to a ring counter which has six or twelve stages.
- One stage is on at a time with the pulse train output of the VCO changing the on stage of the ring counter.
- As each stage turns on it produces a short output pulse once per cycle.
- Over one cycle a complete set of 6 or 12 output pulses are produced by the ring counter at equal intervals.
- These pulses are transferred to the firing pulse generator to the appropriate valves of the converter bridge



- Under steady state conditions V_2=0 and the voltage V_1 is proportional to the AC line frequency ω_1.
- This generates pulses at the line frequency and constant firing delay angles α.
- If there is a change in the current order , margin settings or line frequency., a change in V_3 occurs which in turn results in change in the frequency of the firing pulses.
- A change in the firing delay angle results from the time integral of the differences between the line and firing pulse frequencies.
- It is apparent that this equidistant pulse control firing scheme is based on pulse frequency control.

❖ **PULSE PHASE CONTROL (PPC):** In this scheme a step change in control signals causes a spacing of the only pulse to change these results in a shift of phase only.

ADVANTAGES:

- Equal delay for all the devices.
- Non characteristics harmonics are not introduced

DISADVANTAGES:

- Less DC output voltage than IPC

8-1 SUMMARY

Converters generate harmonic voltages and currents on both ac and dc sides. A converter of pulse number p generates harmonics principally of orders

$$h = pq \tag{1}$$

on the dc side and

$$h = pq \pm 1 \tag{2}$$

on the ac side, q being any integer. Most HV dc converters have pulse number 6 or 12 and thus produce harmonics of the orders given in Table 1. The

Table 1. Orders of Characteristic Harmonics

Pulse No.	DC Side	AC Side
p	pq	$pq \pm 1$
6	0, 6, 12, 18, 24, . . .	1, 5, 7, 11, 13, 17, 19, 23, 25 . . .
12	0, 12, 24, . . .	1, 11, 13, 23, 25 . . .

amplitudes of the harmonics decrease with increasing order: the ac harmonic current of order h is less than I_1/h, where I_1 is the amplitude of the fundamental current.

Unless measures are taken to limit the amplitude of the harmonics entering the ac network and the dc line, some of the following undesirable effects may occur: overheating of capacitors and generators, instability of the converter control, and interference with telecommunication systems, especially noise on telephone lines. These effects may not be confined to the vicinity of the converter station but may be propagated over great distances. The most difficult of these to eliminate is telephone interference.

The principal means of diminishing the harmonic output of converters are (a) increase of the pulse number and (b) installation of filters. High pulse numbers have been used in some converters, but it is the general opinion that for HV dc converters the use of filters is more economical than increase of the pulse number beyond 12. Filters are nearly always used on the ac side of converters. Ac filters serve the dual purpose of diminishing ac harmonics and supplying reactive power at fundamental frequency. On the dc side, the dc reactor diminishes harmonics, and, in many converters, especially those connected to dc cables, no additional filtering is required on the dc side. Dc filters are required, however, on some overhead dc lines.

8-2 CHARACTERISTIC HARMONICS

Definitions and Assumptions

The *pulse number* of a converter is the number of nonsimultaneous commutations per cycle of fundamental alternating voltage.

The *order of a harmonic* is the ratio of its frequency to the fundamental (lowest) frequency of a periodic wave. The order of harmonics on the dc side of a converter, however, is defined with respect to the fundamental frequency on the ac side.

Characteristic harmonics are those of orders given by Eqs. (1) and (2) in Section 8-1.

Noncharacteristic harmonics are those of other orders.

Assumptions. The following assumptions are made as bases for deriving the orders, magnitudes, and phases of the characteristic harmonics of a six-pulse converter:

1. The alternating voltages are three-phase, sinusoidal, balanced, and of positive sequence.
2. The direct current is absolutely constant, that is, without ripple. Such current would be the consequence of having a dc reactor of infinite inductance.
3. The valves are ignited at equal time intervals of one-sixth cycle, that is, at constant delay angle α measured from the zeros of the respective commutating voltages. By assumption 1, these zeros are equally spaced in time.
4. The commutation inductances are equal in the three phases.

Deductions from the Foregoing Assumptions

From assumptions 1 and 2 immediately follow deductions 1 and 2, respectively:

1. The alternating voltage has no harmonics except the first (the fundamental).

2. The direct current has no harmonics.

There can be higher harmonic currents on the ac side and harmonic voltages on the dc side, however, and deductions are made concerning these. Because of assumptions 1, 3, and 4:

3. The overlap angle is the same for every commutation.

4. The ripple of the direct voltage has a period of one-sixth that of the alternating voltage.

5. Hence the harmonics of the direct voltage are of order 6 and its multiples 12, 18, 24, etc.

6. The alternating currents of the three phases have the same wave shape but are displaced by one-third cycle in time (120° of the fundamental).

7. The alternating currents have positive and negative parts of the same shape except that one is inverted; that is, $F(\theta + 180°) = -F(\theta)$.

8. As a result of deduction 7, there are no even harmonics in the alternating current.

9. As a result of deduction 6 and the fact that the phase difference for the hth harmonic is h times that for the fundamental, the ac harmonics have the following phase sequences:

Sequence	Orders (h)
Zero (0)	0, 3, 6, 9, 12, 15, 18, 21, 24, ..., $3q$
Positive (1)	1, 4, 7, 10, 13, 16, 19, 22, 25, ..., $3q + 1$
Negative (2)	2, 5, 8, 11, 14, 17, 20, 23, 26, ..., $3q - 1$

Harmonic analysis of the wave shape of the alternating current shows that

10. No characteristic harmonics of order $3q$ (triple harmonics) can exist.

Final Conclusions on Orders of Characteristic Harmonics. By deduction 5, the direct voltage has only harmonics of orders that are multiples of 6, that is, of orders $6q$, where q is an integer.

By deductions 8 and 10, the alternating currents have only odd harmonics of orders not multiples of 3. Those of orders $6q + 1$ have positive sequence, and those of orders $6q - 1$ have negative sequence.

AC Harmonics at No Overlap

The wave shapes of alternating voltages and currents conforming to the assumptions made are shown in Figure 1. The current waves drawn in solid lines are for any ignition delay angle α but no overlap. The broken curved

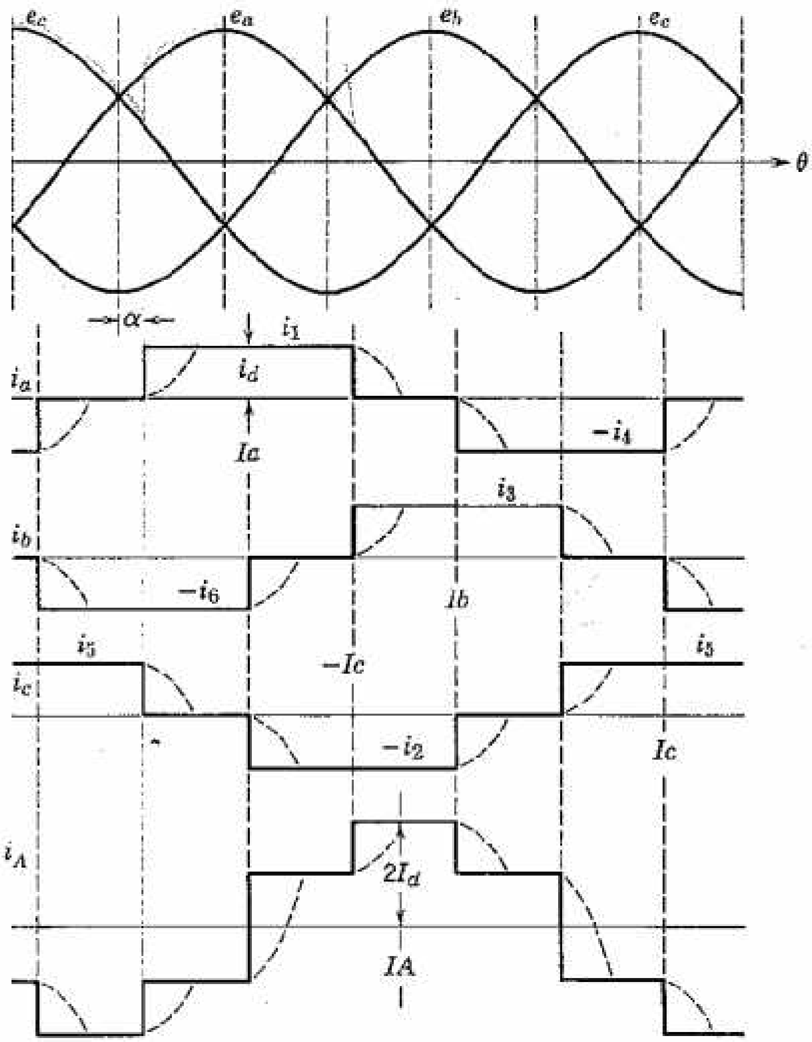


Fig. 1. Wave forms in a six-pulse bridge: line-to-neutral voltages e_a, e_b, e_c and line currents i_a, i_b, i_c with YY-connected transformer; also line current I_A with ΔY-connected transformer.

lines show qualitatively how overlap would modify the fronts and tails of the current pulses.

Valve Currents and Line Currents on Valve Side. The line-current wave forms at no overlap are a series of equally spaced rectangular pulses, alternately positive and negative. Fourier analysis of such a wave shape, for finding the characteristic alternating-current harmonics in this limiting case, is very simple; it also serves to illustrate several features of these harmonics. However, let us take an even simpler starting point: the analysis of a train of positive rectangular pulses of unit height and arbitrary width w radians, that is, of duration w/ω sec (see Figure 2). These pulses might represent the current through one valve.

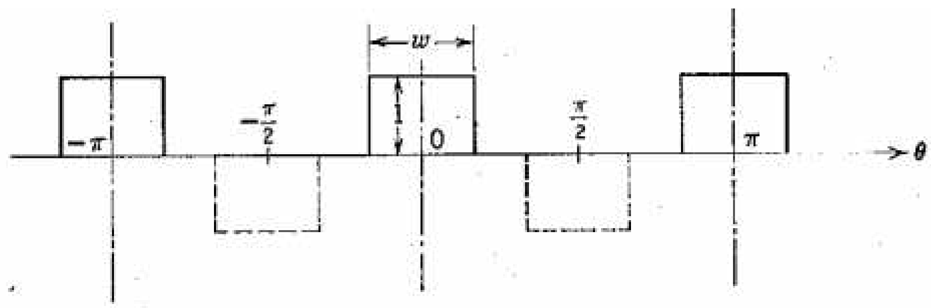


Fig. 2. Trains of positive and negative rectangular pulses of arbitrary width w.

The general trigonometric form of the Fourier series is

$$F(\theta) = \frac{A_0}{2} + \sum_{h=1}^{\infty} (A_h \cos h\theta + B_h \sin h\theta) \tag{3}$$

where

$$A_0 = \frac{1}{\pi}\int_0^{2\pi} F(\theta)\, d(\theta) \tag{4}$$

$$A_h = \frac{1}{\pi}\int_0^{2\pi} F(\theta) \cos h\theta \, d\theta \tag{5}$$

$$B_h = \frac{1}{\pi}\int_0^{2\pi} F(\theta) \sin h\theta \, d\theta \tag{6}$$

The limits of integration in Eqs. (4), (5), and (6) can be taken more generally as σ and $\sigma + 2\pi$, where σ is any angle. $A_0/2$ is the average value of the function F; A_h and B_h are rectangular components of the hth harmonic. The corresponding phasor is

$$A_h - jB_h = C_h \underline{/\phi_h} \tag{7}$$

where

$$C_h = \sqrt{A_h^2 + B_h^2} = \text{crest value}$$

and

$$\phi_h = \tan^{-1} \frac{-B_h}{A_h}$$

If, in the analysis of the wave shown in Figure 2, the origin of θ is taken at the center of a pulse, $F(\theta)$ is an "even" function, and $B_h = 0$ for all h; that is, the series has only cosine terms. Their amplitudes are found by Eq. (5) thus:

$$A_h = \frac{1}{\pi}\int_{-\pi}^{\pi} F(\theta) \cos h\theta \, d\theta = \frac{1}{\pi}\int_{-w/2}^{+w/2} \cos h\theta \, d\theta$$

$$= \frac{1}{\pi h}\left[\sin \frac{hw}{2} - \sin\left(-\frac{hw}{2}\right)\right] = \frac{2}{\pi h} \sin \frac{hw}{2} \tag{8}$$

Also

$$\frac{A_0}{2} = \frac{1}{2\pi}\int_{-w/2}^{+w/2} d\theta = \frac{w}{2\pi} \tag{9}$$

The series is therefore

$$F_1(\theta) = \frac{2}{\pi}\left(\frac{w}{4} + \sin\frac{w}{2}\cos\theta + \frac{1}{2}\sin\frac{2w}{2}\cos 2\theta + \frac{1}{3}\sin\frac{3w}{2}\cos 3\theta + \frac{1}{4}\sin\frac{4w}{2}\cos 4\theta + \cdots\right) \tag{10}$$

In general, this series has a constant term and cosine terms of every harmonic frequency. For certain pulse widths, however, certain cosine terms vanish. This occurs if

$$\frac{hw}{2} = q\pi \qquad \text{or} \qquad w = \frac{2q\pi}{h} \tag{11}$$

For example, the pulses of valve current in the three-phase bridge current have width

$$w = \frac{2\pi}{3}$$

so that if $h = 3, 6, 9, \ldots, 3q$, $\sin(hw/2) = \sin q\pi = 0$. Then the series lacks the third harmonic and its multiples, called triple harmonics for brevity.

Now, if we consider the negative pulses only, shown by broken lines in Figure 2, we get

$$F_2(\theta) = \frac{2}{\pi}\left(-\frac{w}{4} + \sin\frac{w}{2}\cos\theta - \frac{1}{2}\sin\frac{2w}{2}\cos 2\theta + \frac{1}{3}\sin\frac{3w}{2}\cos 3\theta - \frac{1}{4}\sin\frac{4w}{2}\cos 4\theta + \cdots\right) \tag{12}$$

This result can be obtained in at least two ways: (a) by putting the new function into Eqs. (4), (5), (6) and performing the indicated operations or (b) by appropriate changes in series 10. These changes are the following: (1) Shift the pulse π radians; this shifts the fundamental component π rad and shifts the higher harmonics by $\pm h\pi$ rad. If h is even, $\cos(\theta \pm h\pi) = \cos\theta$; but if h is odd, $\cos(\theta \pm h\pi) = -\cos\theta$. Hence the signs of all odd harmonics are changed. (2) Invert the pulse. This changes the sign of every term. The net result is to change the signs of all even-order terms, including the constant term.

Next, let us analyze the train of alternately positive and negative rectangular pulses. Its Fourier series is

$$F_3 = F_1 + F_2 = \frac{4}{\pi}\left(\sin\frac{w}{2}\cos\theta + \frac{1}{3}\sin\frac{3w}{2}\cos 3\theta + \frac{1}{5}\sin\frac{5w}{2}\cos 5\theta + \cdots\right) \quad (13)$$

The constant term and all even harmonics have vanished.

Let us now put $w = 2\pi/3$ and change the height to I_d. For increments of 2 in h, the arguments of the sines increase in increments of $2\pi/3$ rad. For odd h the sines are all $\pm\sqrt{3}/2$, except for triple harmonics, which are zero. The series is

$$i_a = \frac{2\sqrt{3}}{\pi}\cdot I_d\left(\cos\theta - \frac{1}{5}\cos 5\theta + \frac{1}{7}\cos 7\theta - \frac{1}{11}\cos 11\theta + \frac{1}{13}\cos 13\theta - \frac{1}{17}\cos 17\theta + \frac{1}{19}\cos 19\theta - \cdots\right) \quad (14)$$

This contains only harmonics of orders $6q \pm 1$, as predicted earlier. The crest value of the fundamental-frequency current is

$$I_{10m} = \frac{2\sqrt{3}}{\pi} I_d = 1.103 I_d \quad (15)$$

and its effective or rms value is

$$I_{10} = \frac{I_{10m}}{\sqrt{2}} = \frac{\sqrt{6}}{\pi} I_d = 0.780 I_d \quad (16)$$

The effective value of the hth harmonic is

$$I_{h0} = \frac{I_{10}}{h} \quad (17)$$

Series 14 represents the ac line current of phase a on the valve side of the transformer (Figure 1) if the origin of θ is taken at the center of the positive pulse (axis Ia). The currents i_b and i_c in the other two phases have the same wave shape as i_a but are displaced 120° ($= 2\pi/3$ rad) behind and before i_a, respectively. Their Fourier series, if written for $\theta = 0$ at axes Ib and Ic, respectively, are the same as that for i_a written with respect to axis Ia. Likewise, these series are independent of the ignition delay angle α. If any wave is shifted forward by an angle ϕ, measured for the fundamental period, the hth harmonic is shifted by $h\phi$ measured for the shorter harmonic period, being shifted forward if of positive sequence and backward if of negative sequence.

Line Currents on Network Side of Six-pulse Group. If the transformers are connected YY or ΔΔ and have ratios 1 : 1, the line currents on the network side have the same wave shape, hence the same harmonics, as those on the valve side. If, however, the transformers are connected YΔ or ΔY, the wave shape on the network side is different from that on the valve side.

Let the transformers be connected in Y on the valve side and in Δ on the network side, and let the ratio of each individual transformer be 1:1. Then the currents in the delta-connected windings are the same as those in the corresponding Y-connected windings. Each line current on the delta side is the difference of two delta currents; for instance,

$$i_A = i_b - i_c \tag{18}$$

Line current i_A at the bottom of Figure 1 is constructed graphically from the two waves above it. Let us find its Fourier series with respect to $\theta = 0$ at the center of its positive part (axis IA). With respect to this same axis, i_b is retarded 30° and $-i_c$ is advanced 30°. Table 2 shows the magnitude and phase of each harmonic component of these three current waves.

If, instead of the ratio of individual transformers being 1:1, the bank ratio is made 1:1, then the factor $\sqrt{3}$ is removed from every entry in the last column of the table, and the Fourier series becomes

$$i_A = 1.103 I_d(\cos\theta + \tfrac{1}{5}\cos 5\theta - \tfrac{1}{7}\cos 7\theta - \tfrac{1}{11}\cos 11\theta + \tfrac{1}{13}\cos 13\theta + \tfrac{1}{17}\cos 17\theta - \tfrac{1}{19}\cos 19\theta - \cdots) \tag{19}$$

It differs from Eq. (14) only in the signs of the terms representing the fifth, seventh, seventeenth, nineteenth, etc., harmonics. Although all the harmonics in Eqs. (14) and (19) are equal in amplitude, the two series represent different wave shapes because of the difference in signs (or in phase) or certain orders of harmonics.

Alternating Line Currents on Network Side of 12-pulse Converter. A 12-pulse group in a HV dc converter is composed of two 6-pulse groups fed from sets of valve-side transformer windings having a phase difference of 30° (or 90°) between the fundamental voltages. Since α is normally the same for both 6-pulse groups, the fundamental valve-side currents have the same phase difference as the voltages, and the fundamental network-side currents are in phase with one another. The no-load voltage ratios between the network-side windings and each of the two sets of valve-side windings are equal; hence the fundamental network-side currents are also equal.

The resultant network-side current of the two groups is then given by the sum of Eqs. (14) and (19). To keep the power rating of the 12-pulse converter equal to that of a 6-pulse converter, however, both the direct voltage and the alternating current of each of the two bridges of the 12-pulse converter should be half of the corresponding quantities of the comparable one-bridge 6-pulse converter. Half of the sum of Eqs. (14) and (19) is

$$i_{12} = 1.103 I_d \left(\cos\theta - \tfrac{1}{11}\cos 11\theta + \tfrac{1}{13}\cos 13\theta - \tfrac{1}{23}\cos 23\theta + \tfrac{1}{25}\cos 25\theta - \cdots\right) \quad (20)$$

This contains only harmonics of orders $12q \pm 1$. Currents of orders 5, 7, 17, 19, etc., circulate between the two banks of transformers, but do not enter the ac line. If there are two valve windings and one network winding on each transformer, these harmonics appear only in the valve windings. In practice, such harmonics in the two 6-pulse groups are not always exactly equal in magnitude nor exactly in phase opposition; hence their cancellation is incomplete, and they appear to some degree in the network-side line currents. They are *uncharacteristic harmonics* in a 12-pulse converter.

Figure 3 shows the wave shapes of current in each component 6-pulse group and in the 12-pulse group. The fundamental waves are shown in broken lines.

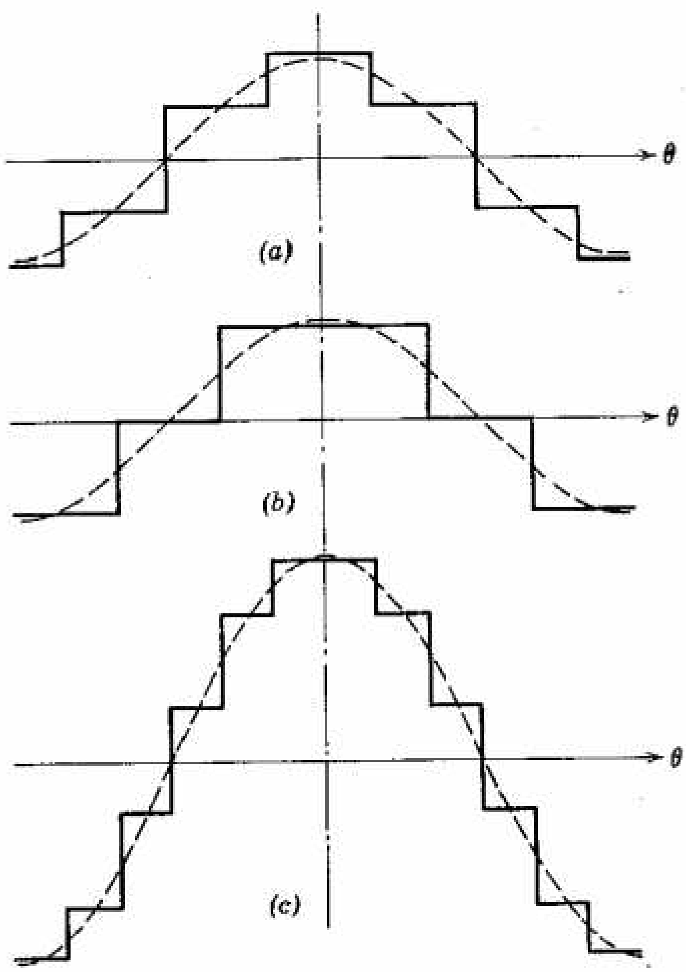


Fig. 3. Alternating line currents of a two-bridge 12-pulse converter with no overlap: (*a*) current of six-pulse bridge with YΔ-connected transformer; (*b*) current of six-pulse bridge with YY-connected transformer; (*c*) total current.

AC Harmonics at Overlap

In Figure 1 the wave shapes for positive overlap appear as better approximations to sine waves than do the wave shapes for no overlap. Hence we make the qualitative deduction that the effect of overlap is to decrease the amplitudes of the harmonics.

Quantitative results are computed from the following formulas. They are valid only for characteristic orders h. For overlap not exceeding 60°, the complex rms value, with phase referred to the respective commutation voltage E is

$$\mathbf{I}_h = K_1 \mathbf{F}_1(\alpha, \delta, h) \qquad \text{amperes} \tag{21}$$

where

$$K_1 = \frac{3}{2\pi h}\left(\frac{E}{X}\right) = \frac{\sqrt{6} I_{s2}}{2\pi h} \qquad \text{amperes} \tag{22}$$

and

$$\mathbf{F}_1 = \frac{\underline{/-(h+1)\alpha} - \underline{/-(h+1)\delta}}{h+1} - \frac{\underline{/-(h-1)\alpha} - \underline{/-(h-1)\delta}}{h-1} \tag{23}$$

Sometimes it is convenient to express the harmonic as a fraction of one of the following currents:

$I_{s2} = \sqrt{3/2}E/X$ = crest value of ac component of current in line-to-line short circuit on valve side.

$I_{\text{base}} = (\sqrt{6}/\pi)I_{s2}$ = rms fundamental alternating current corresponding to $I_d = I_{s2}$ with no overlap.

I_{10} = rms fundamental alternating current with no overlap.

$I_{h0} = I_{10}/h$ = rms harmonic current with no overlap.

I_d = direct current.

The results are as follows:

$$\frac{\mathbf{I}_h}{I_{s2}} = K_2 \mathbf{F}_1 \tag{24}$$

where

$$K_2 = \frac{\sqrt{6}}{2\pi h} \tag{25}$$

$$\frac{\mathbf{I}_h}{I_{\text{base}}} = K_3 \mathbf{F}_1 \quad \text{per unit} \tag{26}$$

where

$$K_3 = \frac{1}{2h} \tag{27}$$

$$\frac{\mathbf{I}_h}{I_{10}} = K_4\mathbf{F}_1 \tag{28}$$

where

$$K_4 = \frac{1}{2hD} = \frac{K_3}{D} \tag{29}$$

$$\frac{\mathbf{I}_h}{I_{h0}} = K_5\mathbf{F}_1 \tag{30}$$

where

$$K_5 = \frac{1}{2D} \tag{31}$$

$$\frac{\mathbf{I}_h}{I_d} = K_6\mathbf{F}_1 \tag{32}$$

where

$$K_6 = \frac{\sqrt{6}}{2\pi hD} \tag{33}$$

where

$$D = \cos\alpha - \cos\delta = 2\sin\frac{\alpha+\delta}{2}\sin\frac{u}{2} = I_d' \tag{34}$$

Usually only the magnitude of a harmonic is wanted, the phase being of no interest. Convenient formulas for computation are the following:

$$I_h = 2K_1F_2(\alpha, u, h) \qquad \text{amperes} \tag{35}$$

$$\frac{I_h}{I_{s2}} = 2K_2F_2 = \frac{\sqrt{6}F_2}{h} \tag{36}$$

$$\frac{I_h}{I_{\text{base}}} = 2K_3F_2 = \frac{F_2}{h} \tag{37}$$

$$\frac{I_h}{I_{10}} = 2K_4F_2 = \frac{F_2}{hD} \tag{38}$$

$$\frac{I_h}{I_{h0}} = 2K_5F_2 = \frac{F_2}{D} \tag{39}$$

$$\frac{I_h}{I_d} = 2K_6F_2 = \frac{\sqrt{6}F_2}{hD} \tag{40}$$

where

$$F_2 = \left(\left\{ \frac{\sin[(h-1)u/2]}{h-1} \right\}^2 + \left\{ \frac{\sin[(h+1)u/2]}{h+1} \right\}^2 - 2 \left\{ \frac{\sin[(h-1)u/2]}{h-1} \right\} \left\{ \frac{\sin[(h+1)u/2]}{h+1} \right\} \cos(2\alpha + u) \right)^{1/2} \quad (41)$$

The last equation has the same form as the law of cosines for the length of one side of a triangle in terms of the lengths of the other two sides and the included angle.

Computed results for I_h/I_{10} versus u are plotted in Figures 4 to 11. Results for $I'_h = I_h/I_{\text{base}}$ versus I'_d at the usual value of α or γ, 15°, are plotted in Figure 12.

Overlap Greater than 60°. In the region bounded by $60° < u < 120°$, $\alpha > 30°$, and $\delta < 150°$, Eqs. (21) to (41) apply if α is replaced by α' and δ by δ', where, as before (Eq. (20) of Chapter 4),

$$\alpha' = \alpha - 30° \qquad \delta' = \delta + 30° \qquad u' = u + 60° \quad (42)$$

Direct-voltage Harmonics

A formula for complex values of the harmonics of the direct voltage is the following:

$$\frac{\mathbf{V}_{dh}}{V_{d0}} = \tfrac{1}{2}\mathbf{F}_3(\alpha, \delta, h) \quad (43)$$

and a formula for the rms values is

$$\frac{V_{dh}}{V_{d0}} = F_4(\alpha, u, h) \quad (44)$$

where

$$\mathbf{F}_3 = \frac{\underline{/(h+1)\alpha} + \underline{/(h+1)\delta}}{h+1} - \frac{\underline{/(h-1)\alpha} + \underline{/(h-1)\delta}}{h-1}$$

$$= \frac{\underline{/(h+1)\alpha}(1 + \underline{/(h+1)u})}{h+1} - \frac{\underline{/(h-1)\delta}(1 + \underline{/(h-1)u})}{h-1} \quad (45)$$

$$F_4 = \left[\left(\frac{\cos[(h-1)u/2]}{h-1} \right)^2 + \left(\frac{\cos[(h+1)u/2]}{h+1} \right)^2 - 2\left(\frac{\cos[(h-1)u/2]}{h-1} \right)\left(\frac{\cos[(h+1)u/2]}{h+1} \right) \cos(2\alpha + u) \right]^{1/2} \quad (46)$$

8-3 UNCHARACTERISTIC HARMONICS

The conditions postulated in the foregoing analysis of characteristic harmonics of a converter are never exactly fulfilled in practice. Consequently, not only are the harmonics of characteristic orders slightly changed from their theoretical magnitudes and phases, resulting in small components of opposite phase sequence from their characteristic sequences, but also—and this is more important—harmonics of uncharacteristic orders are produced. Indeed, a converter is likely to produce harmonics of all orders and some dc component on the valve winding of the transformers.

The harmonics of low uncharacteristic orders are normally much smaller than those of adjacent characteristic harmonics in the converter itself. Filters are usually provided for the low characteristic orders, however, and on the network (or line) side of the filters, the uncharacteristic harmonics may be of about the same magnitudes as those of the characteristic harmonics. For high orders, the magnitudes of both characteristic and uncharacteristic harmonics are small and approximately the same, even before filtering. For the high-order characteristic harmonics, the equations presented in Section 8-2 cannot be depended on for accurate results. The magnitudes of these harmonics and of all the noncharacteristic harmonics can be found only by measurement.

Causes

The ignition delay angle of a rectifier is usually measured from a zero of the commutating voltage. If the three-phase alternating voltages are unbalanced, their zeros are not equally spaced, and, consequently, the valves are not fired at equal time intervals. Probably, even with balanced voltages, there is some "jitter" in the electronic circuitry of the current regulator that produces uncharacteristic harmonics. The variation of firing angles from their normal values is usually cited as 1 or 2°. Reeve and Krishnayya[58] state, however, that on the Cross Channel link the variation was $\pm 3°$ for rectifier operation and $\pm 1.5°$ for inversion.

It was shown in Section 5-11 that the combination of high gain and short time constant in the current regulator would cause alternate early and late ignitions. As a result, harmonics of orders $3q$ are produced in the direct voltage, and harmonics of orders $3q \pm 1$ in the alternating currents. These orders are uncharacteristic if q is an odd integer. For example, a third harmonic and its odd multiples appear in the direct voltage, and even harmonics appear in the alternating currents.

Inverters normally operate on C.E.A. control, and unbalanced three-phase voltages can again lead to unequally timed firing. The C.E.A. control has no feedback. As a rule, inverters on C.E.A. control produce smaller uncharacteristic harmonics than does a rectifier on C.C. control.

Another suggested cause of uncharacteristic ac harmonics is interaction of characteristic-harmonic and fundamental currents in nonlinear elements of the power system.[14,29] The theory of modulation shows that such interaction produces sum and difference frequencies, which, in the case in question, are uncharacteristic. This cause appears to be unimportant, because the principal nonlinear elements of a power system are transformers, in which only the small exciting current is affected by the nonlinear relation between current and flux. Of course, transformers do generate harmonics, but there is no evidence that these interact significantly with converters. The same could be said for corona, which is also a shunt nonlinear element.

Amplification of Uncharacteristic Harmonics. Several HV dc terminals on going into service experienced trouble from a low-order uncharacteristic harmonic of large amplitude causing improper operation, and even instability, of the C.C. control. At Lydd[42] it was the third harmonic; at Benmore, the ninth.[52] Analyses of these troubles have led to the following explanation:[55,56,58] The addition of harmonics to the fundamental three-phase voltage waves shifts the times of voltage zeros from the zeros of the fundamental waves alone. These shifts of zeros cause unequally spaced firings of valves, which, in turn, generate uncharacteristic ac harmonics. If any of these current harmonics meets a high impedance, significant voltage harmonics of like orders are produced. It may happen that one of these uncharacteristic alternating-voltage harmonics has the same harmonic order and phase sequence and nearly the same phase as one of the voltage harmonics assumed at the beginning of this explanation. That particular harmonic is amplified by positive feedback.

If the loop gain is high enough, a harmonic oscillation of increasing amplitude is produced: this is instability.

Consequences

Uncharacteristic harmonics (1) increase telephone interference, because it is not feasible to provide adequate filtering of each order of them, and (2) in some instances cause instability of C.C. control, as explained above.

Suppression or Diminution

In the instances of control instability cited above, a three-phase HV ac filter bank was provided for the offending harmonic (third or ninth). Such a filter provides a low shunt impedance at the frequency of the offending harmonic, with the result that a given current of this frequency produces less voltage of the same frequency, and thus the loop gain is reduced to a degree that gives but little amplification of this harmonic; consequently the control becomes stable.

This method is expensive. A modification of the control system, which operates at a low level of power, would be less expensive. For example, filters to block the offending harmonic could be placed in the three-phase low-voltage circuit that provides a replica of the commutating voltages to the control system. No reason is apparent why a shunt filter in this location could introduce any different transfer function into the loop from what a similar filter placed in the HV circuit would.

A better control system is isochronous control such as the phase-locked oscillator described by Ainsworth.[57] This generates a series of equally timed firing pulses that is locked to the correct average delay angle by the current regulator, actuated by the difference between the set and measured values of direct current. This, or an equivalent scheme, for getting equally timed firing of valves should be used if it is desired to diminish uncharacteristic harmonics as much as possible.

Relation of Uncharacteristic Harmonics to Errors in Ignition Angles

Even AC Harmonics. Suppose that, as previously discussed, the ignition times of valves in a six-pulse bridge are alternately late and early. To be more specific, assume that the odd-numbered valves, constituting one half bridge, are ignited early by an angle ε, while the even-numbered valves, constituting the other half bridge, are ignited late by the same angle. The wave shapes of the alternating currents consist of alternate positive and negative pulses of unaltered duration 120°, but the intervals between a positive pulse and the following negative pulse are increased by 2ε from the normal value.

On pages 297 to 300, the Fourier series for trains of positive only, and of negative only, rectangular pulses were derived, and it was shown that, with the correct relationship between the two trains, the odd harmonics are doubled but the even harmonics vanish. The same relationship holds for the nonrectangular pulses observed where there is overlap. In the event of relative displacement 2ε between the two trains, the resultant harmonics can be found by vector addition, with the odd harmonics separated by $2h\varepsilon$ instead of zero and the even harmonics separated by $\pi - 2h\varepsilon$ instead of by π. The sums are respectively 2 cos $h\varepsilon$ and 2 sin $h\varepsilon$ times the respective harmonics of one train. For small $h\varepsilon$ the decrease of magnitude of the odd harmonics is negligible; the even harmonics except those of order $6q$, which do not appear in the individual trains, increase from zero to a nonzero value, which we proceed to estimate.

The ratio of an even harmonic of order h to the fundamental wave at small overlap is

$$\frac{I_h}{I_1} = \frac{2 \sin h\varepsilon}{2h \cos \varepsilon} = \frac{h\varepsilon - \frac{1}{6}(h\varepsilon)^3 + \frac{1}{120}(h\varepsilon)^5 - \cdots}{h[1 - \frac{1}{2}(h\varepsilon)^2 + \frac{1}{24}(h\varepsilon)^4 - \cdots]}$$

$$\cong \varepsilon[1 + \tfrac{1}{3}(h\varepsilon)^2 + \tfrac{2}{15}(h\varepsilon)^4 + \cdots] \cong \varepsilon \qquad \text{radians} \tag{47}$$

For $\varepsilon = 1°$, corresponding to 2° relative shift between the positive and negative pulses, the second and fourth harmonics are each approximately $1/57.3 = 0.0174$ per unit $= 1.74\%$ of the fundamental current. This value is further decreased by overlap.

Triple AC Harmonics. It was shown in Section 8-2 by Eq. (11) that a train of rectangular pulses of normal width (120°) has no triple harmonics. If, however, the pulse width is longer or shorter than normal, triple harmonics are generated. Again let us estimate the magnitude of such harmonics as a function of the angular ignition error. Suppose that the ignitions of two valves connected to the same phase are late by ε and that the other four valves of the bridge ignite on time. Then the alternating current of that phase consists of positive and negative pulses both of which are shorter than normal by ε. The current of the phase leading that one has pulses longer than normal by ε, and the current of the remaining phase has pulses of normal length. Assume zero overlap, so that the series of Eqs. (10) and (12) are applicable. In each of these series and in their sum, the ratio of odd harmonic to fundamental is

$$\frac{I_h}{I_1} = \frac{\sin (hw/2)}{h \sin (w/2)} \tag{48}$$

Now put $w = 2\pi/3 \pm \varepsilon$ and $h = 3q$. Then

$$\frac{I_h}{I_1} = \frac{\sin (q\pi \pm 1.5q\varepsilon)}{3q \sin (\pi/3 \pm \varepsilon/2)}$$

$$= \frac{\sin q\pi \cos 1.5q\varepsilon \pm \cos q\pi \sin 1.5q\varepsilon}{3q[\sin (\pi/3) \cos (\varepsilon/2) \pm \cos (\pi/3) \sin (\varepsilon/2)]}$$

$$= \frac{\sin 1.5q\varepsilon}{3q[(\sqrt{3}/2) \cos (\varepsilon/2) \pm \frac{1}{2} \sin (\varepsilon/2)]} \tag{49}$$

For small ε, $\cos(\varepsilon/2) \cong 1$, $\sin(\varepsilon/2) \cong 0$, $\sin 1.5q\varepsilon \cong 1.5q\varepsilon$, and

$$\frac{I_3}{I_1} \cong \frac{1.5q\varepsilon}{3q\sqrt{3}/2} = \frac{\varepsilon}{\sqrt{3}} = 0.577\varepsilon \tag{50}$$

For $\varepsilon = 1° = 0.0174$ rad, $I_3/I_1 = 0.01 = 1\%$.

Magnitudes of Uncharacteristic Harmonics Found in Field Tests

Measurements of harmonics from the converter at Lydd (at the English end of the Cross Channel link) were made with the filters disconnected, giving the results shown in Table 4. They were obtained by Fourier analyses of oscillograms.

Table 4. Harmonic Currents Measured on AC Side of Converter at Lydd[39,49]
To an arbitrary scale

Order of Harmonic	Converter Blocked	100-A DC 12-pulse Operation	400-A DC 6-pulse Operation
2	2.0	29.7	25.9
3	2.1	9.3	10.2
4	0.3	10.9	21.6
5	6.0	26.4	92.5*
6	1.3	9.2	6.2
7	4.0	16.2	66.5*
8	0.8	31.7	44.3
9	0.1	57.8	23.8
10	0.5	22.3	43.6
11	2.0	119.6*	75.3*
12	0.6	67.9	3.8
13	1.0	21.5*	19.2*
14	0.3	28.4	15.0
15	0.2	17.9	4.4
16	0.04	18.4	11.1
17	0.2	13.4	7.5*
18	0.3	10.4	3.4
19	0.08	8.6	5.4*
20	0.1	11.7	4.9

* Characteristic harmonics.

8-4 TROUBLES CAUSED BY HARMONICS[38]

List of Troubles

Troubles in the Converter and on the AC Power System

1. Extra losses and heating in machines and capacitors
2. Overvoltages due to resonance
3. Interference with ripple control systems
4. Inaccuracy or instability of the constant-current control of converters

Troubles on the Telecommunication Systems

5. Noise on voice-frequency telephone lines

Noise on telephone lines is the most difficult trouble to eliminate and forms the subject of most of the rest of this chapter. There are reasons for concern, however, over the effects of harmonics on the power system itself. Item 4 was discussed in Section 8-3, page 318. Items 1, 2, and 3 are briefly discussed below.

Extra Losses and Heating in Machines[7,38,50]

Harmonic currents in induction and synchronous machines cause additional losses in, and heating of, these machines. These effects are chiefly attributable to the harmonics of low orders, which can have large magnitudes. Low-resistance damper windings serve to shield the rotor iron from harmonic fluxes that would overheat it, and the heating of the damper windings is small. Damper windings are used on salient-pole synchronous machines but not on round-rotor synchronous machines.

Large harmonics in induction motors reduce the torque available from them at rated speed and cause parasitic torques at lower speeds that can prevent a motor that is being started from attaining its rated speed.

There have been proposals for feeding rectifiers from generators isolated from the rest of the ac system, and this is done at Volgograd. Among the advantages claimed are that the provision of ample damper windings on the generators would cost less than the provision of ac harmonic filters and that the generators could be allowed to vary in speed more than generators usually do in an interconnected ac system with no concern about detuning of these filters. Studies of this possibility on the Nelson River project[59] showed, however, that filtering was more economical than building the generators to carry the harmonic currents continuously. An additional complication was that, because the project would be built by stages, additional generating plants being connected to the rectifiers at each stage, there was no assurance that the harmonics would divide among all these plants in any definite ratio without putting undue restrictions on the design of the future generators, ac lines, etc.

Although filters are provided at the rectifier station, the frequency generated by the plants connected to this station is subject to large variations during system disturbances. The filters are then out of tune, and a greater part of the harmonic current output of the rectifiers passes through the generators, which, however, were designed, at small additional cost, to withstand the temporary additional heating caused by these increased harmonic currents.

Measurements of losses caused by harmonics in a cylindrical-rotor synchronous generator of normal design, loaded by nonlinear reactors were reported by Easton.[50] Measurements of harmonic impedances of a 27.5-MVA generator were reported by Gardiner.[51a]

Extra Losses and Heating in Capacitors

The increase of losses in capacitors due to harmonics is[9,24]

$$\sum_{h=2}^{\infty} C(\tan\delta)\omega_h V_h^2 \tag{51}$$

where C = capacitance

$\tan\delta$ = loss factor

$\omega_h = 2\pi$ times frequency of hth harmonic

V_h = rms voltage of hth harmonic

The dielectric stress is proportional to the crest voltage, which may be either raised or lowered by the harmonic voltages.

The total reactive power, including fundamental and harmonics,

$$Q = \sum_{h=1}^{\infty} Q_h \tag{52}$$

should not exceed the rated reactive power of the capacitor.

Overvoltages from Resonance

The following method may be used to estimate the possibility of resonance between a large shunt capacitor bank on the power system and the rest of the system at a harmonic frequency.[48]

Let Q_s equal short-circuit power of power system at point where capacitor bank is connected, Q_C equal rating of capacitor bank and h equal order of harmonic at which resonance may occur. Then

$$h = \left(\frac{Q_s}{Q_C}\right)^{1/2} \tag{53}$$

Trouble from such resonance is most likely at a frequency close to a harmonic frequency for which no filter is provided. For example, third-harmonic resonance could occur if $Q_C \cong 0.1 Q_s$. Such resonance could have several undesirable effects: (a) overheating of the capacitors, (b) overvoltage at the capacitor bank, and (c) instability of the constant-current regulator of a converter.

Interference with Ripple Control Systems[47]

Some electric-power utilities sell electric energy at especially low rates for off-peak loads, such as water heaters, and control the hours during which such loads can be connected by transmitting audiofrequency tones, in the range of 290 to 1650 Hz, from substations over power-distribution circuits to customers' premises to control contactors in series with such loads. Similar control is used for street lighting by some utilities. The receiving devices for the control signals are broadly tuned and can accept harmonics from high-power converters, which may cause undesired operation of the contactors or prevent desired ones.

The remedies are adequate filtering of ac harmonics or decreasing the susceptibility of the ripple control system to harmonics.

8-5 DEFINITIONS OF WAVE DISTORTION OR RIPPLE

A complete description of a periodic current or voltage wave that is neither constant nor sinusoidal would require either an oscillogram or a list of the harmonics present, together with their phases and magnitudes, or, at least, their magnitudes.

For some purposes, such as filter design, it is convenient to have a more concise expression—a single number—serving as an index of the degree of departure of the wave from its ideal shape. Several such indices have been defined.

Total RMS Harmonics

For alternating current, this quantity is

$$H_1 = \frac{\sqrt{\sum_{h=2}^{\infty} I_h^2}}{I_1} = \frac{\sqrt{I^2 - I_1^2}}{I_1} \qquad \text{per unit} \tag{54}$$

and for direct current it is

$$H_2 = \frac{\sqrt{\sum_{h=1}^{\infty} I_h^2}}{I_d} = \frac{\sqrt{I^2 - I_d^2}}{I_d} \qquad \text{per unit} \tag{55}$$

where I = effective (rms) current
I_d = average direct current
I_1 = rms fundamental current
I_h = rms harmonic current of order h

Similar expressions hold for voltages.

Because the harmonics are squared, the largest ones, that is, the lower orders, dominate the result.

Wasserab[32] calls these indices *Welligkeit* (waviness).

Deviation from a Sine Wave (or a Constant)

For alternating current, it is

$$H_3 = \frac{|i - i_1|_{\max}}{I_{1m}} \quad \text{per unit} \tag{56}$$

and for direct current

$$H_4 = \frac{|i - I_d|_{\max}}{I_d} \quad \text{per unit} \tag{57}$$

where i = instantaneous current
i_1 = instantaneous value of fundamental current wave
I_{1m} = crest value of fundamental wave
I_d = average direct current

In words, the index is the maximum difference of ordinates of the wave in question and of the corresponding fundamental wave or average value. The result depends on the phase position of the harmonics. All harmonics have equal weights.

A closely related index is the *peak-to-peak value of ripple* of direct current or voltage. The maximum positive and negative deviations are taken separately, and their difference is found. Again, it is usually divided by the average value:

$$H_5 = \frac{i_{\max} - i_{\min}}{I_d} \quad \text{per unit} \tag{58}$$

Maximum Theoretical Deviation from a Sine Wave

In many cases the magnitudes of harmonics are known or calculated, but their phase angles are not known. For this reason, it is usually more practical to assume that the crests of all the harmonics occur simultaneously at least once per cycle. The index thus modified is the arithmetical sum of the crest values of all the harmonics divided by the crest value of the fundamental wave. Of course, rms values can be used instead of crest values in both numerator and denominator, giving

$$H_6 = \frac{\sum_{h=2}^{\infty} I_h}{I_1} \tag{59}$$

This index weighs all harmonics equally. It is not used for dc quantities. In practice, we can neglect harmonic orders above the twenty-fifth with very little error.

Psophometric and C-message Weightings

We now come to a group of wave-form indices in which the various frequencies or harmonic orders are weighted according to their effectiveness in interfering with telephone conversations. The sensitivity of the human ear, the response of the telephone receiver, and the coupling between power and telephone circuits all vary with frequency, and these variations can be taken into account by appropriate weighting factors.

Two systems of weighting are in wide use: (a) that of the Bell Telephone System (B.T.S.) and the Edison Electric Institute (E.E.I.) is used in the United States and Canada; (b) another system promulgated by the International Consultative Commission on Telephone and Telegraph Systems (C.C.I.T.T.) is used in Europe. (The C.C.I.T.T. resulted from a merger of C.C.I.F. and C.C.I.T. in 1956.)

One set of weighting factors of each system, based on the sensitivity of the ear and the response of telephone equipment, applies only to currents and voltages on the telephone circuit. This is called *C-message weighting* by B.T.S. and E.E.I., and *psophometric weighting* by C.C.I.T.T. These weights are shown in Tables 5 and 6 and in Figure 18. In both systems maximum weight occurs at 1000 Hz. In the C.C.I.T.T. system, weight 1000 (0 dB) occurs at 800 Hz; in the B.T.S.-E.E.I. system, unit weight occurs at 1000 Hz.

In both systems, the weights have been revised from time to time to reflect the increasing bandwidth and higher standards of quality of telephone transmission. The C-message weighting was adopted in 1960, when it superseded the earlier F1A weighting, which had already superseded the 144-line weighting in 1941. The psophometric weighting presented here is from the 1963 edition of C.C.I.T.T. Directives.[35]

Other sets of weighting factors pertaining to the coupling between power and telephone lines and to the interfering effect of currents and voltages on power systems are described next.

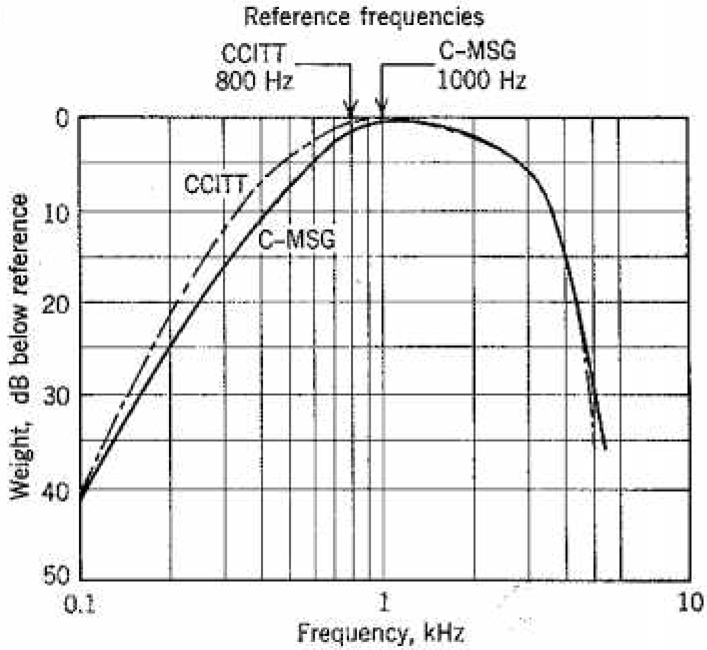


Fig. 18. Comparison of B.T.S. C-message and C.C.I.T.T. weights.

Coupling Factors

In both the B.T.S.-E.E.I. and C.C.I.T.T. systems, the coupling between a power circuit and a telephone circuit is assumed to be directly proportional to frequency. The coupling is taken by convention as 5000 units at 1000 Hz in the B.T.S.-E.E.I. system and as 1 unit at 800 Hz in the C.C.I.T.T. system. Change in coupling as a function of separation or length of circuits is not included in these factors.

Telephone Influence Factor (TIF) and Telephone Harmonic Form Factor

These factors give an approximation to the effect of wave shape of current or voltage of a power line on telephone noise, excluding the magnitude of power-system current or voltage and the geometrical aspects of coupling. Each of these factors is a root-sum-square combination of the products of coupling factors and weighting factors for each frequency. In the B.T.S.-E.E.I. system the result is called *telephone influence factor* (abbreviated TIF), and is defined by

$$\mathrm{TIF} = \frac{\left[\sum_{f=0}^{\infty} (k_f p_f V_f)^2\right]^{1/2}}{V} \tag{60}$$

Table 6. C.C.I.T.T.[35] Psophometric Weights ($1000p_f$) and Telephone Interference Weights fp_f

h	f (Hz)	$1000p_f$	dB	fp_f	h	f (Hz)	$1000p_f$	dB	fp_f
1	50	0.71	−63.0	0.000044	20	1000	1122	+1.0	1.40
2	100	8.91	−41.0	0.00111	24	1200	1000	0.0	1.50
3	150	35.5	−29.0	0.00665	30	1500	861	−1.3	1.62
4	200	89.1	−21.0	0.0223	40	2000	708	−3.0	1.77
6	300	295	−10.6	0.111	50	2500	617	−4.2	1.93
8	400	484	−6.3	0.242	60	3000	525	−5.6	1.97
10	500	661	−3.6	0.413	70	3500	376	−8.5	1.65
12	600	794	−2.0	0.595	80	4000	178	−15.0	0.89
16	800	1000	0.0	1.000	100	5000	15.9	−36.0	0.10

where

$$k_f = 5000(f/1000) = 5f \tag{61}$$

$$p_f = \text{C-message weighting} \tag{62}$$

$$V_f = \text{rms voltage of frequency } f \text{ on power line}$$

$$V = \sqrt{\sum V_f^2} = \text{rms voltage, unweighted} \tag{63}$$

In the C.C.I.T.T. system the result is called *telephone harmonic form factor* (T.H.F.F.) and is defined by a similar expression with

$$k_f = \frac{f}{800} \tag{64}$$

$$p_f = \frac{\text{psophometric weighting}}{1000} \tag{65}$$

Generally the sums are for a finite number of discrete frequencies, which include the power frequency (50 or 60 Hz) and its multiples.

C-message-weighted or Psophometrically Weighted Voltage on the Telephone Circuit

These quantities are either longitudinal or transverse voltages on the telephone circuit weighted psophometrically or by C-message weights (see page 327). They also are root-sum-square combinations of the noise effect of discrete frequencies produced in the telephone by other sources than the speaker's voice. Our attention is limited to voltages induced by a power line.

In the B.T.S.-E.E.I. system, the quantity is called *C-message weighted voltage*. In the C.C.I.T.T. system, the longitudinal induced voltage is called *psophometrically weighted voltage*, and the resulting transverse voltage is called *psophometric voltage*:

$$V_\psi = \sqrt{\sum (p_f V_f)^2} \tag{66}$$

where V_f is the rms longitudinal or transverse voltage of frequency f on the telephone line.

I·T Product, kV·T Product, Equivalent Disturbing Voltage, and Equivalent Disturbing Current

These are weighted currents or voltages in the power systems. In the B.T.S.-E.E.I. system, the *I·T product* is a root-sum-square combination of products of currents (in amperes) of various frequencies, each multiplied by the corresponding TIF:

$$\mathrm{I \cdot T} = \sqrt{\sum (T_f I_f)^2} = I \cdot (\mathrm{TIF}) \tag{67}$$

where I_f = rms current of frequency f
T_f = corresponding single-frequency TIF

The *kV·T product* is a similar combination of power-line voltages (in kV). In the C.C.I.T.T. system, the *disturbing current* or *voltage* is similarly defined. In the use of either system, the power-line current or voltage must be specified as either balanced, that is, positive and negative sequence, or unbalanced, that is, zero sequence.

Analogous quantities on the two systems are tabulated in Table 7.

Table 7. Corresponding Quantities in B.T.S.-E.E.I. and C.C.I.T.T. Systems

B.T.S.-E.E.I.	C.C.I.T.T.
C-message weighting	Psophometric weighting
Telephone influence factor (TIF)	Telephone harmonic form factor (THFF)
C-message-weighted voltage:	
Longitudinal	Psophometrically weighted voltage
Transverse	Psophometric voltage
I·T product	Equivalent disturbing current
kV·T product	Equivalent disturbing voltage

8-6 MEANS OF REDUCING HARMONICS

Increased Pulse Number

In low voltage high-current rectifiers, high pulse numbers have sometimes been used, ranging from 24 to 108. This means of reducing harmonics is very effective as long as all valves are in service, but it requires complicated transformer connections. In HV high-current converters for dc transmission, problems of insulation of the converter transformers to withstand high alternating voltages in combination with high direct voltages dictate simple transformer connections. A pulse number of 12 is easily obtained with simple connections of two six-pulse valve groups, as we have seen, and 24 pulses can be obtained with four six-pulse groups by use of a phase-shifting transformer bank in conjunction with two 12-pulse converters. The required phase shift is 15°. Two ways of obtaining it are shown in Figure 19.

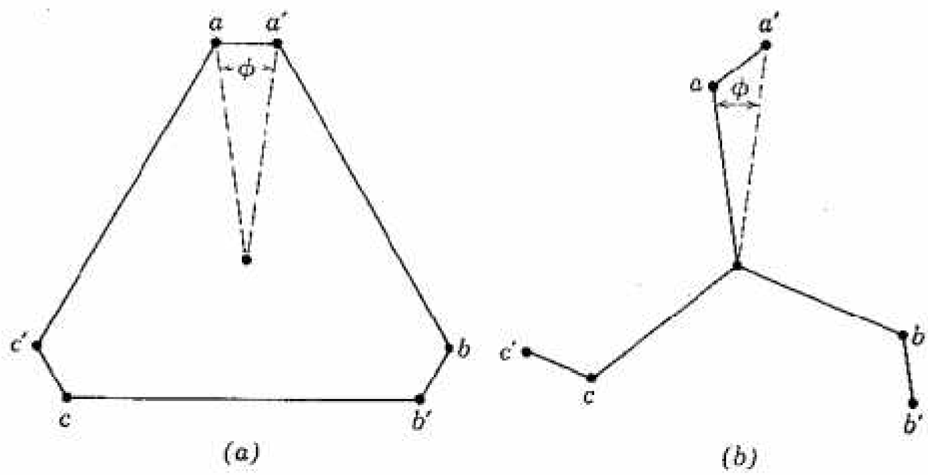


Fig. 19. Voltage vector diagrams of autotransformers for shifting three-phase voltages by angle ϕ (drawn for $\phi = 15°$): *(a)* ring; *(b)* zigzag.

The effectiveness of 12- or 24-pulse converters in reducing harmonics is somewhat decreased when one valve group is out of service. Consider, for example, a converter with four valve groups. If two groups have one transformer connection and the other two groups have a different connection, giving a 30° phase difference, then, with all groups in service, the ac harmonics consist of the 12-pulse harmonics of four groups; but, with only three groups in service, there are the 12-pulse harmonics of three groups and the six-pulse harmonics* of one group. If all groups were connected alike, the harmonic output with three groups in service would consist of both the 12-pulse and six-pulse harmonics of three groups. Obviously, the 12-pulse converter has some advantage over the six-pulse converter even when one bridge is out of service though less than when all are in service.

Filters

Any necessary reduction in harmonic output of the converter beyond that accomplished by increase of pulse number must be done by harmonic filters. Most experts on HV dc transmission feel that it is more economical to use a 12-pulse converter with filters than to use a converter of higher pulse number with the permissible reduction in filters. Therefore, the emphasis in the rest of this chapter is on design of adequate harmonic filters.

Filters are almost always needed on the ac side of the converter and, sometimes, on the dc side also. The ac filters serve two purposes simultaneously: supplying reactive power of fundamental frequency in addition to reducing harmonics. Hence the part of the cost of filters chargeable to the need for reducing harmonics is usually near to the cost of the filter inductors, the filter capacitors being required for supply of reactive power. Thus we are led to the concept of the *minimum filter*, which is required for harmonic reduction only in installations where the reactive power required by the converter can be supplied by the ac system without reinforcing the latter. A filter costing more than the minimum filter not only supplies additional reactive power but also generally gives better filtering. Care should be taken, however, that not too much reactive power is supplied during operation of the dc link at light load.

In addition to the ac harmonic filter at the converter stations, such filters could also be placed in any sections of transmission line giving rise to especially bad telephone noise. This is seldom, if ever, done, because it is usually cheaper to modify or relocate the telephone line.

8-7 TELEPHONE INTERFERENCE

General

The frequencies used in commercial voice transmission range from 200 to 3500 Hz. In this range lie many harmonics of the power-system frequency which are usually of small magnitude but which, because of the high TIF weightings and the great difference between the power levels at which power circuits and telephone circuits operate, may, nevertheless, result in perceptible —or even unacceptable—telephone noise. The power on a voice-frequency telephone circuit is from 10^{-3} to 10^{-5} W. By contrast, that on a power distribution circuit is 10^3 to 10^5 W, and on a major transmission line is 10^7 to 10^9 W.

History

The subject of inductive coordination of power and telephone circuits has been thoroughly studied by both power-system and telephone engineers for the last 50 yr or more. In the United States those studies have been made principally by the B.T.S. and E.E.I.; in Europe, by the C.C.I.F. (now the C.C.I.T.T.) and by governmental power and telephone authorities. Improvements in the quality of telephone service during this period have resulted in a decrease of the tolerable psophometric voltage. Rectifiers in industrial and railway service are not new, but the converters for HV dc transmission have now attained much higher power ratings than those.

Coupling, Electric and Magnetic

Coupling between power circuits and telephone circuits is through both electric and magnetic fields. Unless the spacing between the two circuits is close, however—for example, if both circuits are on the same poles—the magnetic coupling predominates, and the electric coupling is negligible. The magnetic coupling may be expressed as a *mutual impedance*, that is, as the voltage induced in the telephone circuit per ampere of current in the power circuit.

Spacing. The coupling between two circuits having parallel conductors increases with increased spacing between conductors of the same circuit and decreases with increased distance between circuits.

Length. The coupling between parallel circuits is directly proportional to the common length, known as the length of exposure.

Metallic and Ground-return Circuits. If the ground were a perfect conductor, there would be no electric field or varying magnetic field in the ground, and the fields above the surface of the ground caused by overhead conductors carrying currents and charges would be the same as that caused by the actual conductors and their *image conductors* with the ground removed. On the assumption that the surface of the ground is a horizontal plane, each image conductor is a fictitious conductor like the corresponding real conductor, located directly below the latter as far below the surface as the real conductor is above it and carrying current and charge equal in magnitude but opposite in direction to those in the real conductor.

If the ground has finite and uniform conductivity, the foregoing statements are still substantially true with respect to the electric field. The magnetic field, however, can now penetrate the earth, and its effect on self- and mutual inductances is as if the image conductors were lowered to a greater depth below the surface of the ground. The *equivalent depth of ground return* is proportional to the *skin depth*; both of these depths vary inversely as the square root of the frequency and of the conductivity.

If the ground is nonuniform, the foregoing is still true qualitatively.

Because the distance between the overhead conductors of a ground-return circuit and their image conductors are much greater than the distances between conductors of a metallic circuit and because the two conductors of a ground-return circuit are in a vertical plane, the coupling between two ground-return circuits is very much greater than the coupling between two metallic circuits separated by the same distance as the ground-return circuits.

Although ground-return circuits were used for dc telegraphy and voice telephony when these arts were new, they are seldom if every used now because of the severe problems of noise and cross talk. Power circuits are also all metallic except for some HV dc lines, because the power loss and telephone interference from ground-return ac circuits are both high. It would, therefore, appear that the coupling to be calculated is that between a metallic power circuit and a metallic telephone circuit. But, on the contrary, the practice is to calculate the coupling between a ground-return power circuit and an open-ended ground-return telephone circuit. In other words, one calculates the *longitudinal voltage* induced in the telephone circuit by *residual current* in the power circuit. The reasons for this practice are now given.

Unbalances. The currents in a metallic three-phase power circuit are nominally balanced. In telephone parlance *balanced currents* have a sum that is zero. In power-system parlance, the phase currents have positive- and negative-sequence components but no zero-sequence components. In practice, the line-to-ground fundamental-frequency voltages of the power system are almost entirely of positive sequence. The impedances of the three phases, however, are not perfectly balanced because of the inequality of interphase spacing and the lack or infrequency of transpositions. Consequently, some zero-sequence current exists. The power circuit is usually grounded at the source end, and some are multigrounded. At least, there are shunt capacitances to complete the zero-sequence path. The balance of a power circuit seems to be poorer for the higher harmonics than for fundamental currents.

The *residual current*, which, in a three-phase circuit, is $3I_0$, generally induces both longitudinal and transverse voltages in a parallel telephone circuit. If the power and telephone circuits are well separated, however (say, 0.5 km or more) and if the two wires of the telephone circuit are close together and frequently transposed, the induced transverse voltage is negligible, but the induced longitudinal voltage is much greater. This induced longitudinal voltage, however, would still produce no noise in the telephone circuit if the latter were perfectly balanced; but small unbalances in the telephone circuit give rise to unbalanced currents, accompanied by a *transverse voltage*.

Unbalances on the telephone circuit may consist of any unequal series impedances or shunt impedances to ground; for example, ringers connected from one wire to ground and slight differences in resistance of wires or of their capacitances or leakage to ground.

To sum up, the following series of events causes balanced voltages in metallic power systems to produce transverse noise voltage in metallic telephone circuits:

Balanced voltage on power circuit, through
Unbalance of power circuit, causes
Residual current on power circuit, which, through
Coupling between two ground-return circuits, induces
Longitudinal voltage in telephone circuit, which, through
Unbalance of telephone circuit, causes
Transverse voltage in telephone circuit

Shielding (Screening). Passive ground-return circuits between the power and telephone circuits or near to either of them can affect the coupling and the balance. Among such circuits are ground wires on open power lines and metallic sheaths on power cables and on telephone cables, all of which are multigrounded. Their usual effect is to decrease the coupling. Any parallel circuits, however, that are unsymmetrically located with respect to the power circuit and near it also increase the unbalance of the latter. Either the screening effect or the unbalancing effect may predominate. Other parallel, energized power lines can have both of these effects and also be an additional source of telephone influence. For underground cable circuits, the earth has a screening effect, and for submarine cables, the water has an even greater effect.

The *screening factor* is the ratio of the induced telephone noise with the screening to that without the screening.

Frequency. The coupling increases with frequency. For circuits having fixed current paths, the inductive mutual reactance is directly proportional to frequency. Because the equivalent depth of ground return decreases with increasing frequency, however, the coupling between two ground-return circuits increases more slowly than as the first power of the frequency.

The coupling is usually calculated at 1000 Hz (B.T.S.-E.E.I.) or at 800 Hz (C.C.I.T.T.). Then the variation of coupling with frequency is calculated as a separate factor. As previously noted, in the calculation of TIF, the coupling is assumed to vary in direct proportion to the frequency, which is an approximation.

The mutual impedance between two ground return circuits is usually computed by Carson's formula,[1] which assumes a homogeneous earth and parallel conductors. Krakowski[54] has extended Carson's work to crossing conductors. Riordan and Sunde[4,11] have extended this work to a two-layer earth, that is, to surface layer of one resistivity separated by a horizontal plane from an infinite volume having a different resistivity.

If the separation between power and telephone circuits is much greater than that between conductors of the same circuit, the conductors of each circuit may be replaced by one equivalent conductor at the center of gravity of the several conductors. The coupling between widely separated ground-return circuits increases with the resistivity of the ground.

Unbalance of Telephone Circuit

The *balance factor* of a telephone pair is the ratio of the longitudinal voltage induced in the telephone pair by current in a power circuit to the transverse voltage resulting from unbalance of the telephone circuit. For a subscriber loop in modern cable, a typical value of this factor is said to be 316, or 50 dB.[40] An open-wire circuit would be more poorly balanced and thus have a lower balance factor.

Unbalance of Power Circuit

A common configuration of the three conductors of a three-phase transmission line is for them to be in a horizontal or vertical plane with equal spacing between adjacent conductors. Transpositions are rarely used. If such a line carries balanced (positive-sequence) currents, the zero-sequence voltage drop is approximately one-thirtieth of the positive-sequence voltage drop, corresponding to 30 dB.

If the conductors have a triangular configuration, the balance is better, being theoretically perfect for an equilateral triangle. Transpositions, when used, help to balance the line for the fundamental. For the high harmonics, the number of transposition cycles per wavelength is usually too small to be effective. Ground wires on the power line usually improve the balance slightly, but the ground wires most commonly used have a much higher resistance than do the main conductors; hence, their effect is small.

Direct-current overhead lines of the usual configuration are inherently well balanced.

8-8 HARMONIC FILTERS

Purposes

The ac harmonic filters serve two purposes: (1) to reduce the harmonic voltages and currents in the ac power network to acceptable levels and (2) to provide all or part of the reactive power consumed by the converter, the remainder being supplied by shunt capacitor banks, by synchronous condensers, or by the ac power system. The dc harmonic filters serve only to reduce harmonics on the dc line.

Types

The filters at a converter station may be classified by their location, their manner of connection to the main circuit, their sharpness of tuning, and the number and frequencies of their resonances.

Location. Filters are located on both ac and dc sides of converters. Filters on the ac side may be connected either on the primary (network) side of the converter transformers or on the tertiary winding if one is provided for this purpose. Filters are never connected to the secondary (valve side) windings.

Since the tertiary windings, if provided, have a lower voltage than the primary windings, the filters are insulated for lower power-frequency and surge voltage and, therefore, cost less. The tertiary windings, however, add to the cost of the transformers. These windings usually have a high leakage reactance, which inherently forms a common branch in series with all the shunt filters and complicates the computation of possible resonances between the filters and the ac network.

Series or Shunt. Harmonics may be (a) impeded in passing from the converter to the power network or line by a high series impedance, (b) diverted by a low shunt impedance, or (c) both. Figure 20 illustrates the first two kinds. Each is a dual of the other.

The series filter must carry the full current of the main circuit and must be insulated throughout for full voltage to ground. The shunt filter can be grounded at one end and carries only the harmonic current for which it is tuned plus a fundamental current much smaller than that of the main circuit. Hence, a shunt filter is much cheaper than a series filter of equal effectiveness.

Ac shunt filters have another advantage over series filters in that at fundamental frequency the former supplies needed reactive power but the latter consumes it.

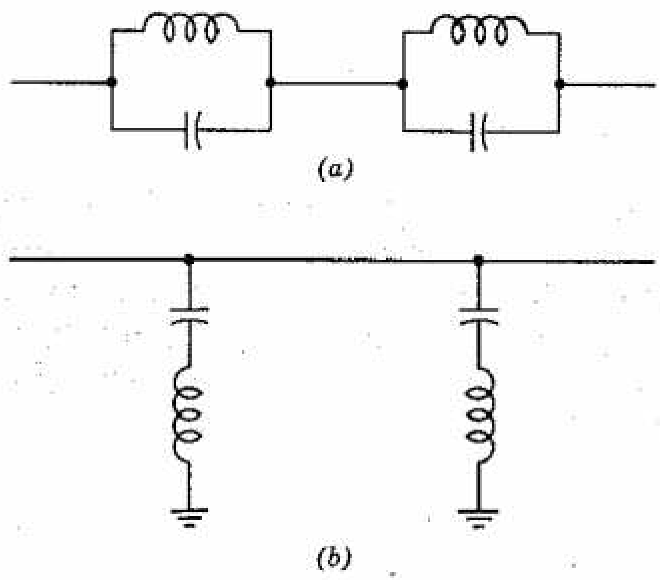


Fig. 20. (*a*) Series filter and (*b*) shunt filter.

For these two reasons shunt filters are used exclusively on the ac side. On the dc side, the dc reactor, which is obviously a series element, constitutes all or part of the dc filters. It must withstand high direct voltage to ground and high direct current. It serves several additional functions (Section 7-2), however, that require series connection. The remainder of the dc filters (if used) consists of shunt branches.

Ac filters could be Δ-connected, but this connection offers no advantage; therefore, the Y connection with grounded neutral is used.

Sharpness of Tuning. Two kinds are used: (a) the *tuned filter* (high Q filter), which is sharply tuned to one or two of the lower harmonic frequencies, such as the fifth and seventh, and (b) the *damped filter* (low Q filter), which, if shunt-connected, offers a low impedance over a broad band of frequencies embracing, for example, the seventeenth and higher harmonics. The second kind is also called a *high-pass filter*. Figures 21 and 22 show typical circuit diagrams and characteristics of the two types. They are analyzed under "Design of Tuned Filter," page 355, and "Design of High-pass Damped Filters," page 375, respectively.

Cost of Filters

The capital cost of ac filters is in the range of 5 to 15% of the cost of the terminal equipment.* This is high enough to justify careful design from the standpoint of economy as well as adequacy. The cost of losses should also be taken into consideration. The cost of filters may be partly charged to reactive-

* For example, the cost of the filters of the New Zealand scheme was said to be 12%.[52]

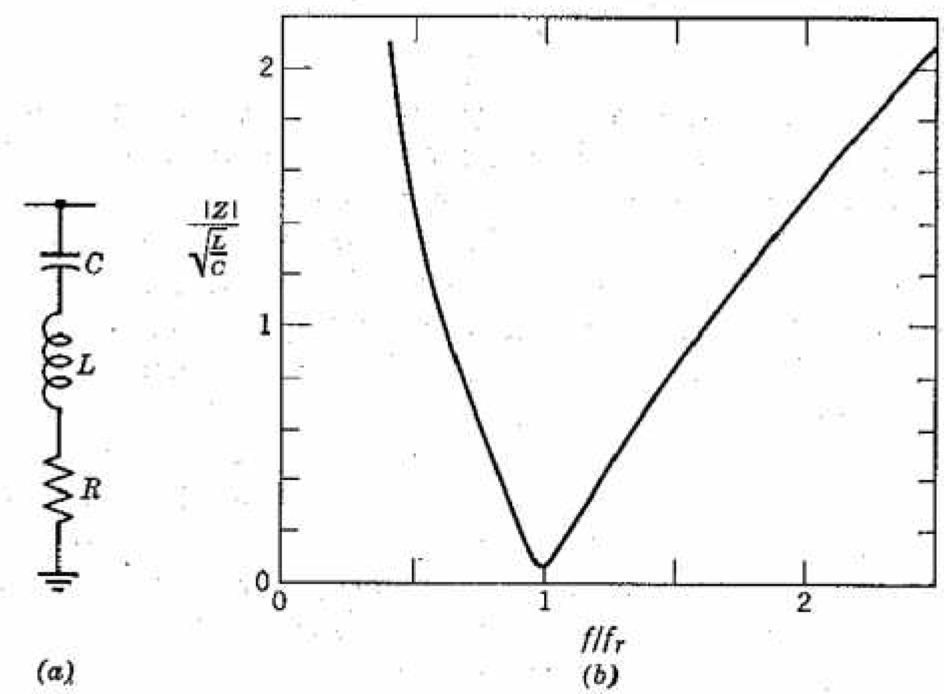


Fig. 21. Single-tuned shunt filter: (*a*) circuit; (*b*) impedance versus frequency.

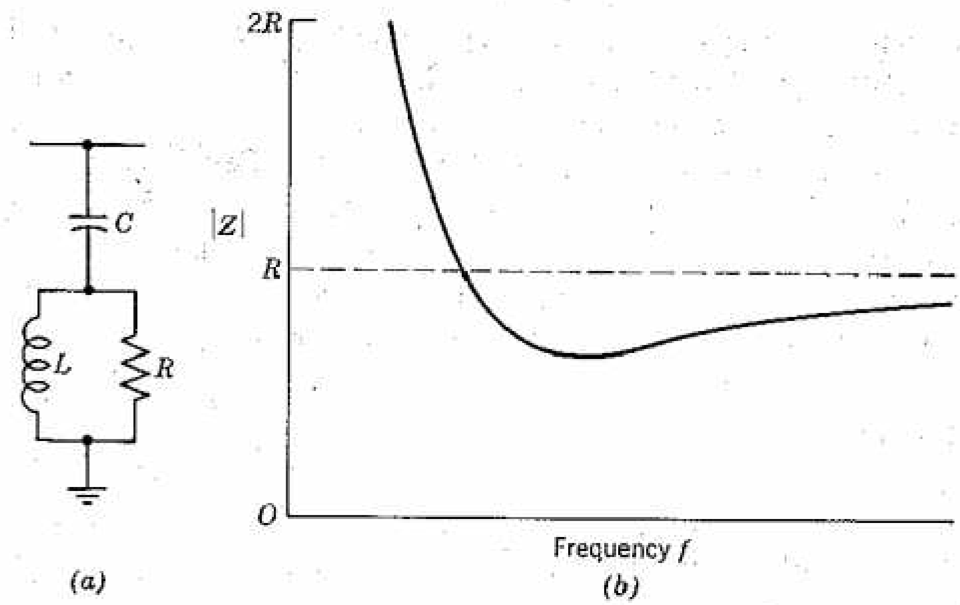


Fig. 22. Second-order damped shunt filter: (*a*) circuit; (*b*) impedance versus frequency.

power supply and partly to filtering though there is no logical basis of the division.

A *minimum filter* is one that adequately suppresses harmonics at the least cost and supplies some reactive power but perhaps not all that is required.

A *minimum-cost-filter* is defined under "Minimum-cost Tuned Filter," page 368. It may or may not give adequate filtering.

About 60% of the capital cost of the filters is that of the capacitors. Hence, substantial savings are possible by judicious choice of kind of capacitor.

Criteria for Adequacy of AC Filters

Ideally, the criterion should be the absence of all detrimental effects from harmonics, including telephone interference, which is the most difficult effect to eliminate entirely. This criterion is impractical from both technical and economic standpoints. From the technical standpoint of filter design, the distribution of harmonics throughout the ac network is too difficult to determine in advance. From the economic standpoint, the reduction of telephone interference can generally be accomplished more economically by taking some of the measures in the telephone system and others in the power system.

The practical criterion would be an acceptable level of harmonics at the converter terminals, expressed in terms of harmonic current, of harmonic voltage, or of both. The filter designer would prefer a criterion based on harmonic voltage at the converter terminals because he can more readily guarantee staying within a reasonable voltage limit than a reasonable current limit despite changes in the network impedance seen from the converter terminals.

Unfortunately, there is no general agreement on the acceptable limit of either harmonic current or harmonic voltage. Presently, we can look only at the limits that have been proposed or attained.

Stumpf[40] stated that, from the experience of the Bell Telephone System with industrial rectifiers, an $I \cdot T$ product greater than 25 kA would be likely to cause severe interference problems; one less than 5 kA would be unlikely to cause any interference problems.

Several others have proposed limits for harmonic voltage.

1. Ainsworth[41] has suggested the following limits:
   a. Maximum theoretical deviation from a sine wave (H6, Section 8-5, page 327) is not to exceed 3 to 5%.
   b. Telephone harmonic form factor (THFF, Section 8-5, page 328) is not to exceed 1 or 2%.
2. Iliceto[38] reported that, for the Sardinian project,
   a. H6 had been specified as 4% (a value said to be satisfactory for turbogenerators, induction motors, and fluorescent lamps) and
   b. Maximum THFF as 1%.
3. The values proposed by C.E.G.B. for the Kingsnorth scheme are the the following[44]:
   a. Limit every single characteristic-harmonic voltage to 1%. This should be calculated with the most unfavorable network impedance within the chosen impedence bounds (Figure 28).

b. Limit the arithmetic sum of the characteristic harmonic voltages of orders 5 to 25 to 2.5% with any one harmonic as in (*a*) and the rest from the most unfavorable impedance locus.

4. Filters for the New Zealand scheme[47] were designed so that each characteristic harmonic would be less than 0.7%.

Effect of Network Impedance on Filtering

The converter approximates a constant-voltage harmonic source on the dc side and a constant-current harmonic source on the ac side. More accurately, the converter is a low-impedance harmonic source on the dc side and a high-impedance harmonic source on the ac side. We now consider, on the ac side, the effect of filter impedance and network impedance on the harmonic voltage V_h at the converter terminals and on the harmonic current I_{hn} entering the network.

Figure 23 shows an equivalent circuit for the purpose. The harmonic

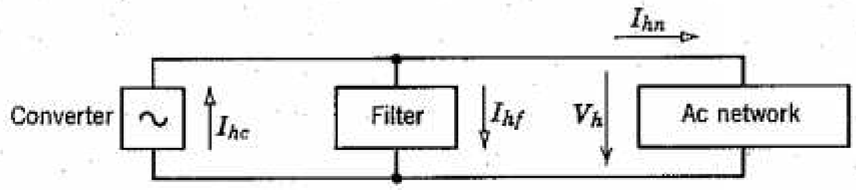


Fig. 23. Circuit for computation of harmonic currents and voltages on the ac side of a converter.

current I_{hc} generated in the converter is assumed to be known. It splits between two paths, the shunt filter and the network. The harmonic voltage across this parallel circuit depends on the impedance of these two branches in parallel. Let

$\mathbf{Z}_{hf}$ = impedance of filter to harmonic of order h
$\mathbf{Z}_{hn}$ = impedance of network to harmonic of order h

$$\mathbf{Y}_{hf} = \frac{1}{\mathbf{Z}_{hf}} \quad \text{and} \quad \mathbf{Y}_{hn} = \frac{1}{\mathbf{Z}_{hn}}$$

Then the harmonic voltage is

$$\mathbf{V}_h = \frac{\mathbf{Z}_{hf}\mathbf{Z}_{hn}\mathbf{I}_{hc}}{\mathbf{Z}_{hf} + \mathbf{Z}_{hn}} = \frac{\mathbf{I}_{hc}}{\mathbf{Y}_{hf} + \mathbf{Y}_{hn}} \tag{69}$$

and the harmonic currents in the network and filter, respectively, are

$$\mathbf{I}_{hn} = \frac{\mathbf{V}_h}{\mathbf{Z}_{hn}} = \frac{\mathbf{Z}_{hf}\mathbf{I}_{hc}}{\mathbf{Z}_{hf} + \mathbf{Z}_{hn}} = \frac{\mathbf{Y}_{hn}\mathbf{I}_{hc}}{\mathbf{Y}_{hf} + \mathbf{Y}_{hn}} \tag{70}$$

$$\mathbf{I}_{hf} = \frac{\mathbf{V}_h}{\mathbf{Z}_{hf}} = \frac{\mathbf{Z}_{hn}\mathbf{I}_{hc}}{\mathbf{Z}_{hf} + \mathbf{Z}_{hn}} = \frac{\mathbf{Y}_{hf}\mathbf{I}_{hc}}{\mathbf{Y}_{hf} + \mathbf{Y}_{hn}} \tag{71}$$

Since the impedance of the network to harmonics is subject to change and is seldom accurately known, the effect of some extreme assumptions is investigated:

1. If the network impedance were nil to all harmonics, there would be $\mathbf{V}_h = 0$ and $\mathbf{I}_{hn} = \mathbf{I}_{hc}$. Shunt filters would have no effect. All the harmonic current generated by the converter would enter the network. Filtering would appear perfect if judged by voltage but bad if judged by current. This assumption of $\mathbf{Z}_{hn} = 0$ is unrealistic. If it were approximately true, filters with series elements would be required.

2. If the network impedance were infinite, all the harmonic current generated by the converter would pass through the filter. There would be $\mathbf{I}_{hn} = 0$, $\mathbf{I}_{hf} = \mathbf{I}_{hc}$, and $\mathbf{V}_h = \mathbf{Z}_{hf}\mathbf{I}_{hc}$. Filtering would be perfect if judged by current and could be good if judged by voltage, for the design of suitable filters would present no great problem. This assumption of $\mathbf{Z}_{hn} = \infty$, although obviously untrue, might give reasonable results as regards harmonic voltages.

3. There is, however, a more pessimistic assumption: that the network and filter are in parallel resonance. The resulting impedance would be a high resistance; and $\mathbf{V}_h$, $\mathbf{I}_{hn}$, and $\mathbf{I}_{hf}$ would all be high. Indeed, the harmonic network current and voltage could be increased by the presence of the filter. The filtering could be bad, whether judged by current or voltage or both. Moreover, the filter could be overloaded; that is, its elements would be subjected to both high harmonic current and high harmonic voltage.

Since tuned filters are customarily provided for the low characteristic harmonics and since the impedance of such a filter at the frequency to which it is tuned is a low resistance, severe parallel resonance of filter and network to such a harmonic is unlikely unless the filter passband is too narrow and unless either the system frequency is abnormal or the filter is detuned. Such resonance is likewise unlikely at the higher frequencies for which the high-pass damped filter provides a low impedance and high power factor. It is more likely to occur at a low uncharacteristic harmonic. It is unlikely to occur at more than one harmonic frequency at the same time although, because of changes in the network, it could occur at another harmonic frequency at another time.

The severity of resonance depends on the amount of damping due to losses both in the filters and in the network. Therefore, some knowledge of the response of the network to harmonics is desirable.

Impedance of the AC Network

The impedance of the ac network seen from the converter terminals, as a function of frequency, may be either measured or calculated. Both methods offer certain difficulties.

Measurement. Measurements must be made with the power system alive at high voltage. Since the ac network contains other sources of harmonics, measurements of network impedance at harmonic frequencies require a high-power source of harmonics. An adjustable-speed motor-generator set may be used for generating a single frequency, adjustable, perhaps, from 180 to 1200 Hz. Alternatively, a rectifier short-circuited on the dc side can be used to generate all its characteristic harmonics simultaneously. Either source may be single-phase and may be connected to the ac network through a step-up transformer. For greatest usefulness, the complex impedance must be measured, not merely the scalar value.

Measurements apply only to conditions at the time of measurement and not to future conditions under which a converter may operate. Nevertheless, measurements give some useful information that cannot be obtained by computation. Results of measurements are discussed later.

Calculation. Calculation can be done by either a model (network analyzer) or a digital computer. Calculations can be made for light and heavy loads, outages of lines or equipment, and planned future conditions. The principal uncertainties are due to inadequate knowledge of the circuit parameters at harmonic frequencies and to the effect of unbalances of the circuits.

The following are some suggestions for details of the power-system representation:

Represent only the positive-sequence network.

Overhead lines may be represented by one equivalent π section, whose branches are corrected for each frequency, or, alternatively, by several nominal π circuits in tandem.

Transformers may be represented by a fixed leakage inductance in series with a resistance that is a function of frequency. Their capacitances are neglected.

Generators are represented by an inductance equal to between 0.8 and 0.9 of their subtransient inductances.

Loads may be represented by resistances in series with the transformer inductances.

In the use of a network analyzer, it is more feasible to use a constant-frequency source and to readjust the circuit impedances for each harmonic than to use an adjustable-frequency source with a fixed network.

Examples and Conclusions. Three loci of network impedance in the complex impedance plane for the 220-kV Italian mainland network at the San Dalmazio terminal of the Sardinia dc link are given in Figures 24, 25, and 26.

These loci were measured on a model. They illustrate several points:

1. Alternation of resonance (low resistance) and antiresonance (high resistance) as the frequency increases.

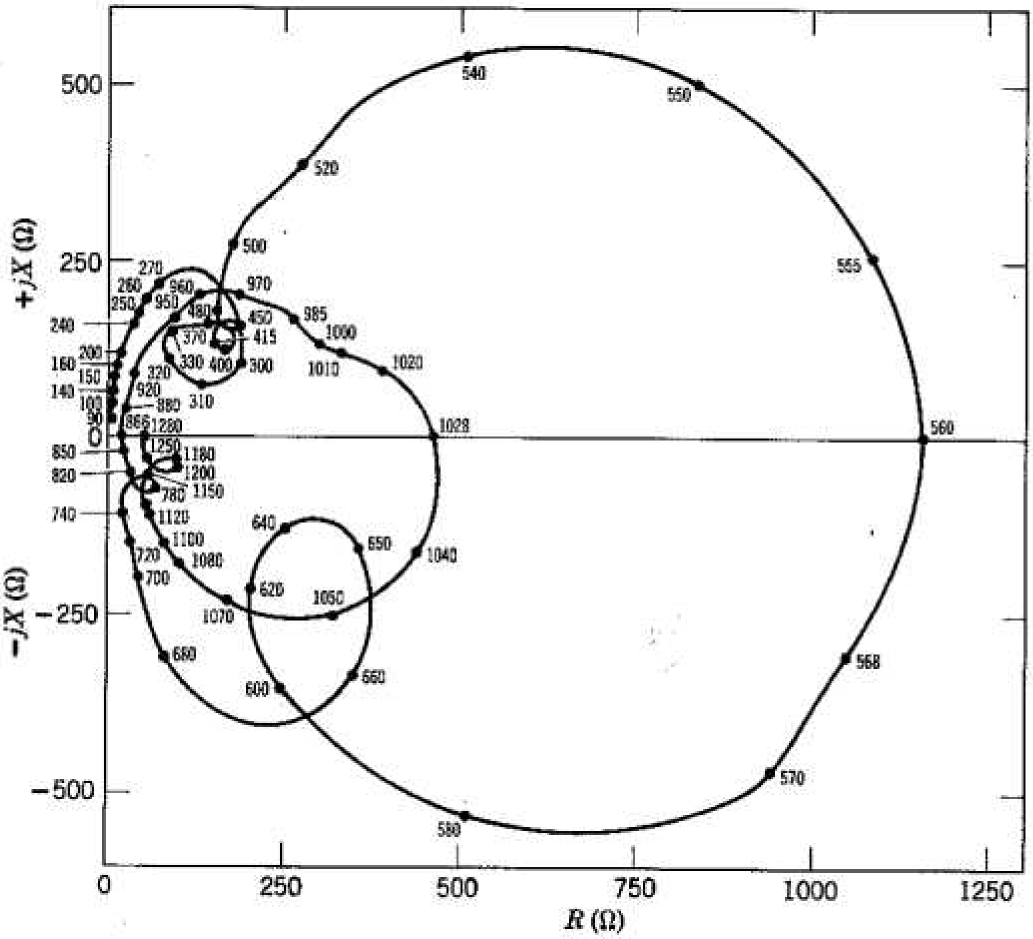


Fig. 26. Calculated impedance of 220-kV network at minimum load with two lines disconnected. (From Ref. 38 by permission.)

AC Filter Design—General Remarks

Aims are (a) to achieve adequate harmonic reduction and (b) to supply the required reactive power at fundamental frequency, (c) achieving both at minimum cost.

Composition. The ac filters in each phase usually comprise:

1. Tuned filters for several (2 to 8) lower harmonics
2. A damped filter for higher harmonics
3. Switchable shunt capacitors

The lower characteristic harmonics have the largest current magnitudes and, therefore, require filters that have low impedances at and near the frequencies of these harmonics. It is more economical to use a separate tuned branch for each of these harmonics than to provide a wide-band filter of sufficiently low impedance.

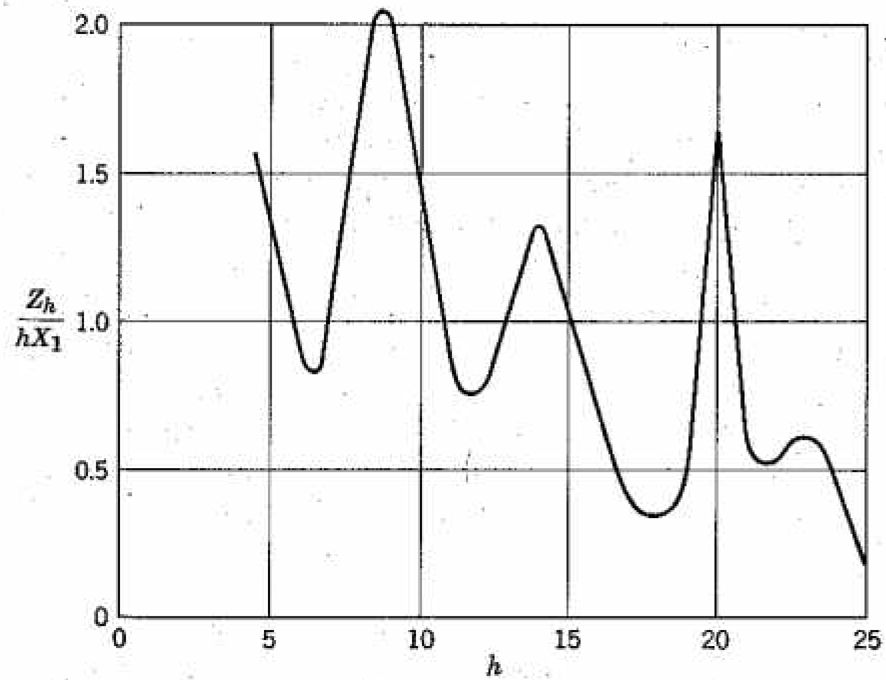


Fig. 27. Measured harmonic impedance of the 132-kV 50-Hz system at Lydd. (From Ref. 49 by permission.)

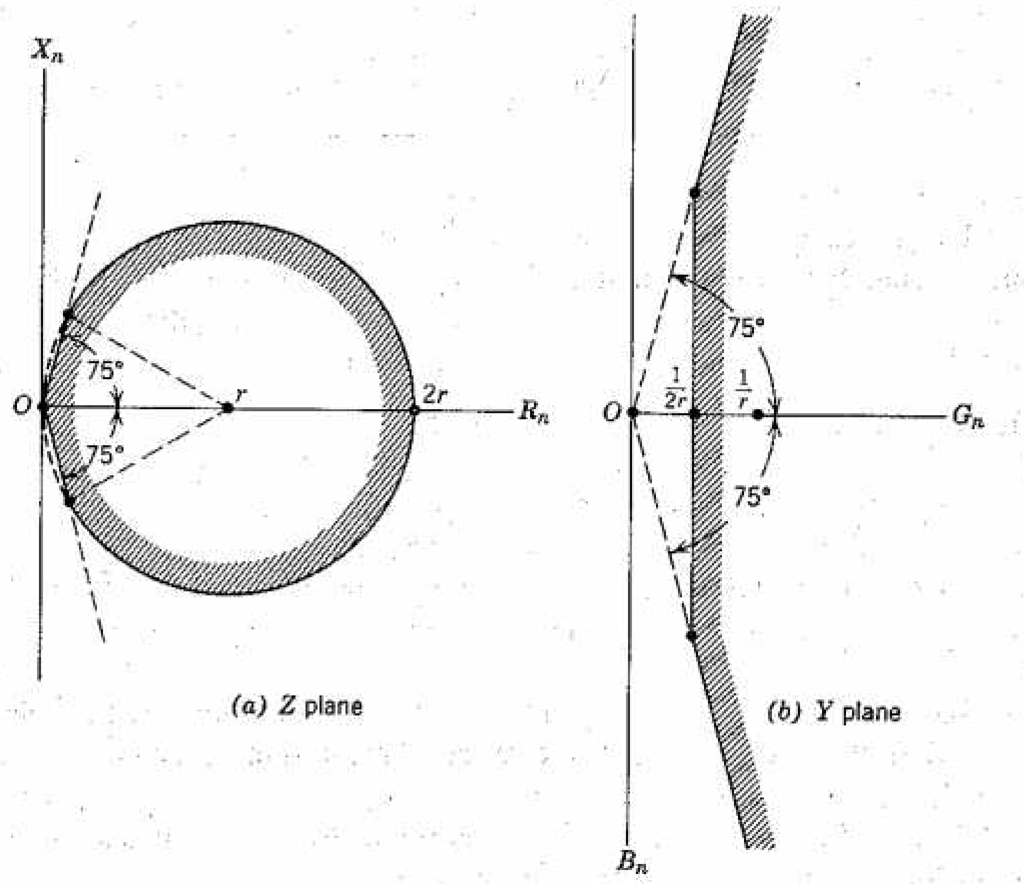


Fig. 28. Bounds of loci of (*a*) impedance and (*b*) admittance to harmonics in an ac network.

The higher harmonics have smaller magnitudes, and the frequency ratio of successive harmonics approaches unity. A great many tuned filters would be required, and their passbands would overlap anyhow. A damped high-pass filter is more economical for these higher harmonics.

The number of tuned filter arms varies from one dc link to another, the commonest number being four. Logically, the combination of tuned arms and high-pass arms should be the cheapest one that satisfies the filtering requirements. Provision of tuned filters for the seventeenth and nineteenth harmonics may depend on the number of bridges in the converter.

The relative magnitudes of characteristic harmonics of 2- and 4-bridge 12-pulse converters are shown in Table 8 for all bridges in service and for one

Table 8

Number of Bridges	Harmonic Current of Order h in Per-unit of Full-Load Fundamental Current								
	$h = 1$	5	7	11	13	17	19	23	25
2/2 or 4/4	1.000	0	0	0.091	0.077	0	0	0.044	0.040
3/4	0.750	0.050	0.036	0.068	0.058	0.015	0.013	0.033	0.030
1/2	0.500	0.100	0.071	0.046	0.038	0.029	0.026	0.022	0.020

bridge out of service. Filters for the fifth, seventh, seventeenth, and nineteenth harmonics are needed only when a bridge is out of service.

Shunt capacitors are used mainly for varying the reactive power when the load on the converter changes. They also improve the filtering of high harmonics.

Size. The size of a filter is defined as the reactive power that the filter supplies at fundamental frequency. It is substantially equal to the fundamental reactive power supplied by the capacitors. The total size of all the branches of a filter, including shunt capacitors, is determined by the reactive-power requirement of the converter station and by how much of this requirement can be supplied by the ac network and by synchronous condensers, if any.

The size of individual arms depends on filtering requirements, but seldom is it less than the size for minimum cost (see "Minimum-cost Tuned Filter," page 368).

The design of tuned filters involves selection of their size and sharpness of tuning (Q), and is discussed immediately below.

The design of high-pass damped filters involves selection of their size, sharpness of tuning, and resonant frequency; it is discussed beginning on page 375.

Design of Tuned Filters

Single-tuned Filters. A single-tuned filter is a series *RLC* circuit (Figure 21) tuned to the frequency of one harmonic (generally a low characteristic harmonic). Its impedance is given by

$$\mathbf{Z}_f = R + j\left(\omega L - \frac{1}{\omega C}\right) \tag{72}$$

At its resonant frequency, its impedance is a low resistance R. Its passband is commonly regarded as bounded by frequencies at which $|\mathbf{Z}_f| = \sqrt{2}R$. At these frequencies the net reactance equals the resistance, and the impedance angle is $\pm 45°$.

Let the quantities ω, R, L, C in Eq. (72) be replaced by the following:

$$\omega_n = \frac{1}{\sqrt{LC}} = \text{tuned angular frequency (rad/sec)} \tag{73}$$

$$\delta = \frac{\omega - \omega_n}{\omega_n} = \text{deviation (per unit) of frequency from tuned frequency} \tag{74}$$

$$X_0 = \omega_n L = \frac{1}{\omega_n C} = \sqrt{\frac{L}{C}} = \text{reactance of inductor or capacitor (ohms) when } \omega = \omega_n \tag{75}$$

$$Q = \frac{X_0}{R} = \text{quality factor of inductor or sharpness of tuning of filter (dimensionless)} \tag{76}$$

From these,

$$\omega = \omega_n(1 + \delta) \tag{77}$$

$$C = \frac{1}{\omega_n X_0} = \frac{1}{\omega_n R Q} \tag{78}$$

$$L = \frac{X_0}{\omega_n} = \frac{RQ}{\omega_n} \tag{79}$$

Substitution of Eqs. (77), (78), (79) into Eq. (72) gives

$$\mathbf{Z}_f = R\left(1 + jQ\delta\frac{2 + \delta}{1 + \delta}\right) \tag{80}$$

For the small frequency deviations ($\delta \ll 1$) in which we are now interested, the impedance is given very nearly and more simply by

$$\mathbf{Z}_f \cong R(1 + j2\delta Q) = X_0\left(\frac{1}{Q} + j2\delta\right) \tag{81}$$

$$|\mathbf{Z}_f| \cong R\sqrt{1 + 4\delta^2 Q^2} = X_0\sqrt{Q^{-2} + 4\delta^2} \tag{82}$$

The admittance, conductance, and susceptance under like conditions are

$$\mathbf{Y}_f \cong \frac{1}{R(1+j2\delta Q)} = \frac{1-j2\delta Q}{R(1+4\delta^2 Q^2)} = \frac{Q-j2\delta Q^2}{X_0(1+4\delta^2 Q^2)} \tag{83}$$

$$|\mathbf{Y}_f| \cong \frac{1}{R\sqrt{1+4\delta^2 Q^2}} = \frac{Q}{X_0\sqrt{1+4\delta^2 Q^2}} \tag{84}$$

$$G_f \cong \frac{1}{R(1+4\delta^2 Q^2)} = \frac{Q}{X_0(1+4\delta^2 Q^2)} \tag{85}$$

$$B_f \cong \frac{2\delta Q}{R(1+4\delta^2 Q^2)} = \frac{2\delta Q^2}{X_0(1+4\delta^2 Q^2)} \tag{86}$$

Inductive susceptance is positive; capacitive, negative.

Frequency Deviation (Detuning). In practice a filter is not always tuned exactly to the frequency of the harmonic that it is intended to suppress.

1. The power-system frequency may change, thus causing the harmonic frequency to change proportionally.
2. The inductance of the inductor and the capacitance of the capacitor may change. Of these two, the capacitance changes more because of aging and change of temperature due to ambient temperature and self-heating (see "Capacitors," page 365).
3. The initial tuning may be off because of finite size of tuning steps.

A change of L or C of 2% causes the same detuning as a change of system frequency of 1%. The total *detuning* or *equivalent frequency deviation* is, consistent with Eq. (74),

$$\delta = \frac{\Delta_f}{f_n} + \frac{1}{2}\left(\frac{\Delta L}{L_n} + \frac{\Delta C}{C_n}\right) \tag{87}$$

In subsequent analysis, δ is assumed to be wholly attributable to Δf.

Graphs of Impedance. Figure 29 shows three curves of filter impedance $|\mathbf{Z}_f|$ versus frequency deviation δ. Curves A and B are for the same R; they have the same minimum impedance. Curves B and C are for the same X_0; they have the same asymptotes D (corresponding to $R = 0$). The equation of the asymptotes is $|X_f| = \pm 2X_0|\delta|$. Curves A and C are for the same Q; th.y have the same passband **PB**. From Eq. (81) the edges of the passband are at $\delta = \pm 1/2Q$, and the width of the passband is $1/Q$.

From these curves it is apparent that the impedance of the filter at its resonant frequency can be decreased by decreasing R. In order to keep the impedance low over a frequency band bounded by the points of maximum

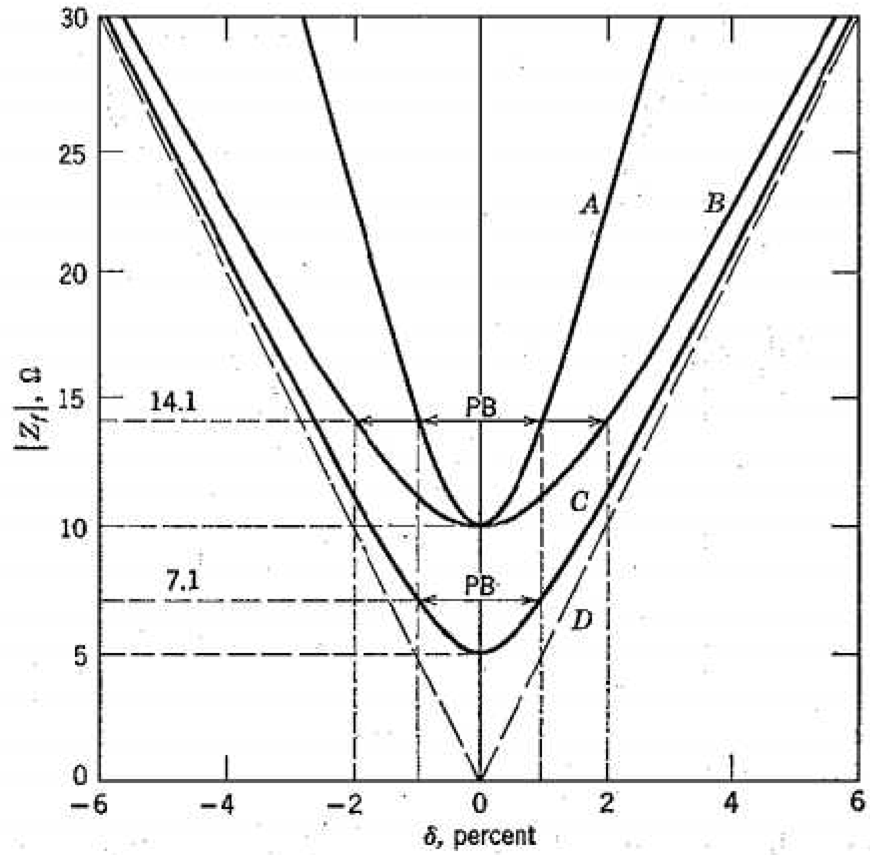


Fig. 29. Impedance of tuned filter as function of frequency deviation. Curve *D* consists of the asymptotes of curves *B* and *C*.

Curve	*R* (Ω)	X_0 (Ω)	*Q*	Passband (PB)
A	10	500	50	2%
B	10	250	25	4%
C	5	250	50	2%

expected frequency deviation, however, it may be necessary to decrease X_0 also, thereby decreasing Q.

Figure 30 has a generalized dimensionless impedance curve with coordinates $y = |\mathbf{Z}_f|\, Q/X_0$ versus $x = Q\delta$. In these coordinates, the minimum impedance is 1, the width of the passband is 1, and the asymptotes are $y = \pm 2|x|$. The curve is a hyperbola, given by $y^2 = 4x^2 + 1$.

Minimization of Harmonic Voltage V_h (see "Effect of Network Impedance on Filtering," page 347) requires minimization not of the filter impedance $\mathbf{Z}_{hf}$ alone but of the impedance $\mathbf{Z}_h$ resulting from the parallel combination of filter impedance $\mathbf{Z}_{hf}$ and the impedance $\mathbf{Z}_{hn}$ of the ac network—(Eq. 69) and Figure 23):

$$V_h = |\mathbf{V}_h| = |\mathbf{Z}_h|\,\mathbf{I}_{hc} = \frac{|\mathbf{I}_{hc}|}{|\mathbf{Y}_h|} = \frac{|\mathbf{I}_{hc}|}{|\mathbf{Y}_{hf} + \mathbf{Y}_{hn}|} \tag{88}$$

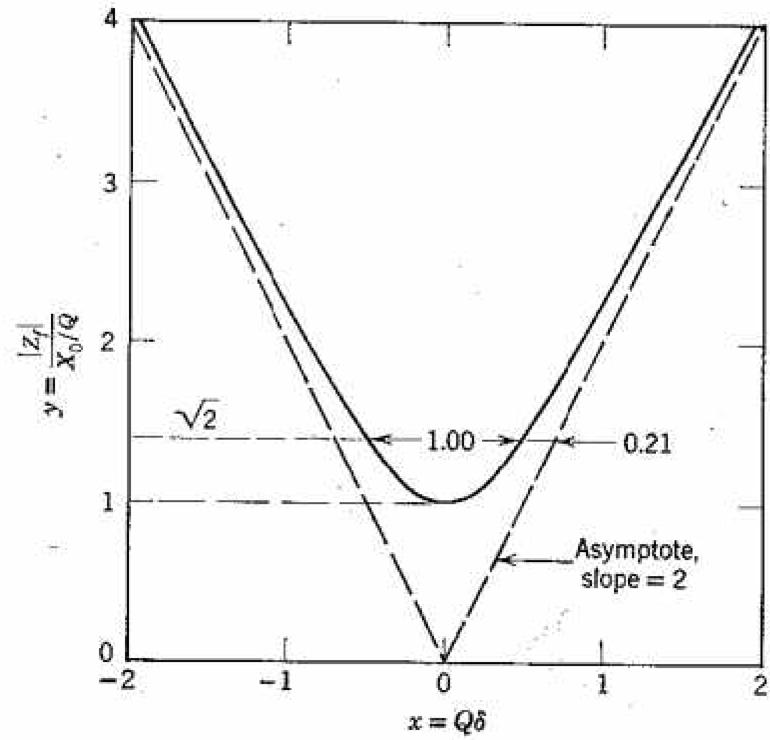


Fig. 30. Generalized impedance curve of tuned filter.

The variables that are not under the control of the filter designer are chosen pessimistically; that is, so as to give the highest V_h. Then the variables that are under his control are chosen optimally to give acceptable V_h. The variables for which pessimistic values are assumed are the frequency deviation δ and the network impedance $\mathbf{Z}_{hn}$. Harmonic voltage is shown to increase with δ; hence, the pessimistic value of δ is the greatest value that is expected to persist: δ_m. The network impedance is taken as the worst value within reasonable limits (see "Impedance of the AC Network," page 348). The variables that the designer can vary, within reasonable limits, are the Q and the "size" of the filter. There is an optimum value of Q that gives minimum harmonic voltage for the assumed network conditions, and this value, denoted by Q_o should be used. It is independent of filter size. Then size is chosen for acceptable harmonic voltage and for the desired amount of reactive power. Since Q_o depends on the assumptions about the network impedance, several cases must be examined.[41]

Case 1. Infinite Network Impedance. In this case the resultant impedance is merely that of the filter: $\mathbf{Z}_h = \mathbf{Z}_{hf}$. By substitution of Eq. (82) for $|\mathbf{Z}_f|$ into Eq. (88), the harmonic voltage is given as

$$V_h = |\mathbf{Z}_{hf}|I_{hc} = I_{hc}X_0(Q^{-2} + 4\delta_m^2)^{1/2} \tag{89}$$

For given X_0 and δ_m, V_h is minimized by making

$$Q = Q_o = \infty \tag{90}$$

The harmonic voltage is then

$$V_h = 2\delta_m X_0 I_{hc} \tag{91}$$

In practice there is a maximum Q for which a coil of given inductance can be built to operate at a given frequency, and economy dictates a somewhat lower Q. If the harmonic voltage is unacceptably high at this Q, it becomes necessary to decrease X_0 by increasing the size of the filter.

The assumption of infinite network impedance is optimistic and unrealistic, because it rules out the possibility of resonance between the network and the filter, which increases the harmonic voltage.

Case 2. Purely Reactive Network. We now pass to consider the most pessimistic assumption about the network. Equation (88), with admittances expressed in terms of their components, becomes

$$V_h = \frac{I_{hc}}{\sqrt{(G_{hf} + G_{hn})^2 + (B_{hf} + B_{hn})^2}} \tag{92}$$

In the present case we may put $G_{hn} = 0$ and also, on the assumption of resonance, $B_{hf} + B_{hn} = 0$. Then, simply,

$$V_h = \frac{I_{hc}}{G_{hf}} \tag{93}$$

and substitution of Eq. (85) for G_{hf}, with $\delta = \delta_m$, gives

$$V_h = X_0 I_{hc}(Q^{-1} + 4\delta_m^2 Q) \tag{94}$$

This is minimized if

$$Q = Q_0 = \frac{1}{2\delta_m} \tag{95}$$

giving the harmonic voltage as

$$V_h = 4\delta_m X_0 I_{hc} \tag{96}$$

which is twice the value—Eq. (91)—obtained in case 1.

The present case is unduly pessimistic, because every power network has some conductance that decreases the voltage at parallel resonance.

Case 3. Network with Limited Impedance Angle. Let the network impedance angle ϕ be limited to values between $\pm\phi_m$, where $0 < \phi_m < 90°$. It is shown that the highest harmonic voltage occurs if $\phi = \phi_m$ and has opposite sign from that of δ. Since no limit was placed on $|\mathbf{Y}_{hn}|$, we must find and use the value that minimizes $|\mathbf{Y}_h|$ and, hence, maximizes V_h. Here a graphical analysis is informative.

As before, the greatest value of δ, δ_m, must be assumed, and optimum Q must be found, this being the value that maximizes $|\mathbf{Y}_h|$.

Figure 31*a* shows the locus of filter impedance—Eq. (81)—

$$\mathbf{Z}_{hf} = X_0(Q^{-1} + j2\delta_m),$$

with fixed $X_f = 2\delta_m X_0$ and variable $R_f = X_0/Q$ as a horizontal line in the $\mathbf{Z}$ plane. In the $\mathbf{Y}$ plane (part *b* of the figure) this line, inverted, becomes a semicircle of diameter $1/(2\delta_m X_0)$ tangent to the G axis at the origin. Points on the semicircular locus in the $\mathbf{Y}$ plane, corresponding to points on the rectilinear locus in the $\mathbf{Z}$ plane for the same values of Q, are found by drawing radial lines from the origin of each plot with equal but opposite slopes; for example, the points for $Q = 1/(2\delta_m)$ lie on lines of slope ± 1 (angle $\pm 45°$). Vectors from the origins to points on the loci represent filter impedance and admittance, $\mathbf{Z}_{hf}$ and $\mathbf{Y}_{hf}$, respectively.

In Figure 31*b* vector $\mathbf{Y}_{hf}$ is tentatively taken as that for $Q = 1/(2\delta_m)$, and a tentative vector $\mathbf{Y}_{hn}$ is added to it to give $\mathbf{Y}_h$. The terminal points of $\mathbf{Y}_{hn}$ and

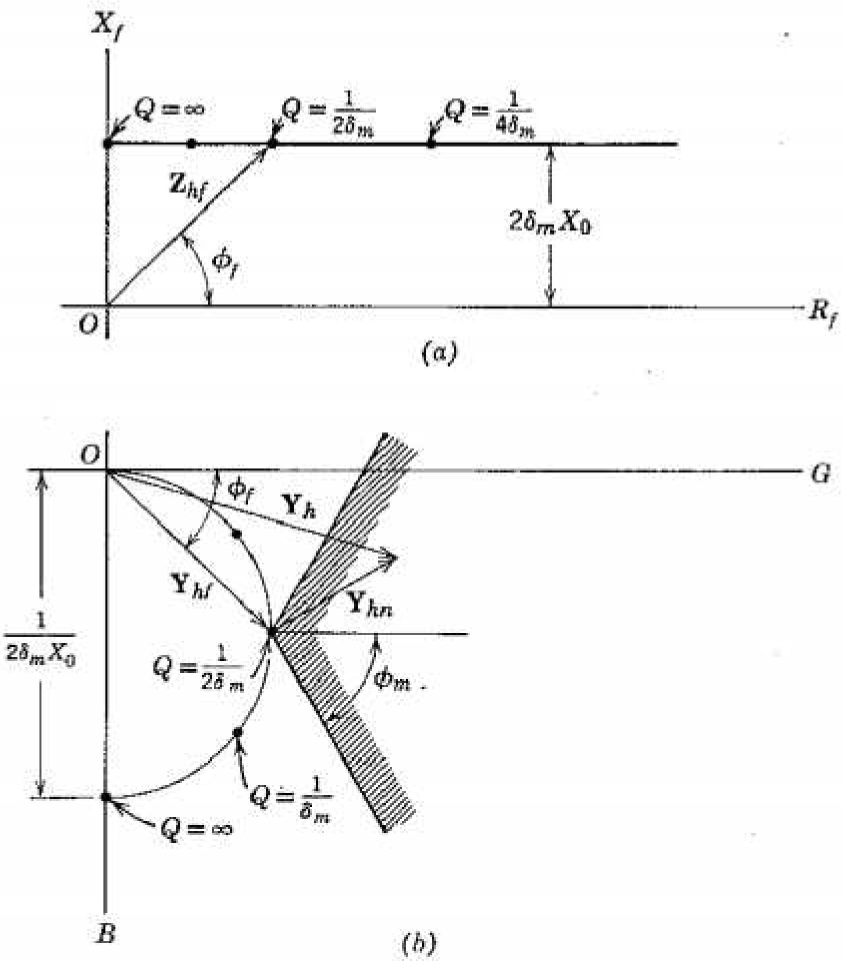


Fig. 31. Loci of (*a*) filter impedance $\mathbf{Z}_{hf}$ and (*b*) filter admittance $\mathbf{Y}_{hf}$ with constant X and varying R and Q; (*b*) also shows tentative choices of $\mathbf{Y}_{hf}$ and $\mathbf{Y}_{hn}$.

$\mathbf{Y}_h$ must lie within or on the boundary of the shaded area, drawn there for $\phi_m = 60°$. It is readily seen that the tentative choice of vectors does not give minimum $\mathbf{Y}_h$. For the assumed $\mathbf{Y}_{hf}$ the shortest vector $\mathbf{Y}_h$ is perpendicular to the boundary and terminates on the boundary above the vertex. Furthermore, it may be seen that the tentative choice of $\mathbf{Y}_{hf}$ is not that which maximizes $\mathbf{Y}_h$ with respect to Q. The proper $\mathbf{Y}_{hf}$ is that which ends on the semicircle at a point where the boundary at angle $+\phi_m$ is tangent to the semicircle.

In Figure 32 the vectors are redrawn so that $\mathbf{Y}_{hf}$ maximizes $\mathbf{Y}_h$, and $\mathbf{Y}_{hn}$

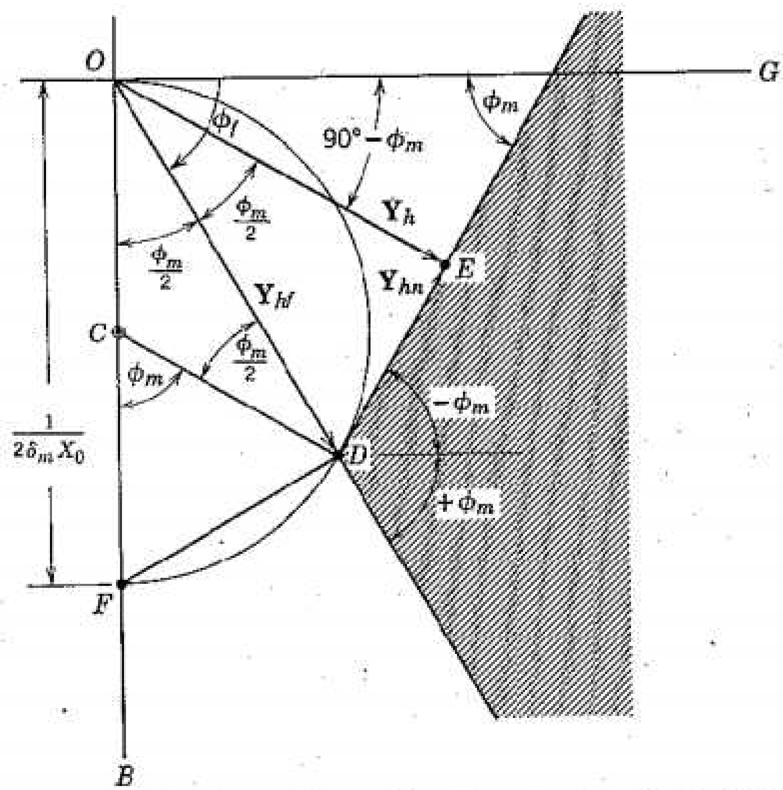


Fig. 32. Construction for finding optimum Q and worst network admittance $\mathbf{Y}_{hn}$. Drawn for $\phi_m = 60°$.

minimizes it. The truth of the statement made earlier, that $|\phi| = \phi_m$ and that the sign of ϕ is opposite to the sign of δ, is proved by the vector diagram, drawn for positive $\delta = \delta_m$ and for negative $\phi = \phi_m$. (Remember that ϕ was defined as an impedance angle and that $-\phi$ is the corresponding admittance angle.) In addition, the effect of varying δ may be shown. Decrease of δ increases the diameter of the semicircular locus of $\mathbf{Y}_{hf}$, thereby increasing $\mathbf{Y}_h$ and decreasing V_h. Hence $|\delta| = \delta_m$ is more pessimistic than $|\delta| < \delta_m$. Negative δ turns the figures upside down.

Formulas for optimum Q and for the corresponding $\mathbf{Y}_{hf}$, $\mathbf{Y}_{hn}$, and $\mathbf{Y}_h$ can be derived by trigonometry, starting with the known diameter of the semicircle, the known angle ϕ_m, and other angles related to ϕ_m. Triangle

OCD is isosceles, with the two smaller angles $\angle COD = \angle CDO = \phi_m/2$. In right triangle ODF, $OD = OF \cos(\phi_m/2)$; hence

$$|\mathbf{Y}_{hf}| = \frac{\cos(\phi_m/2)}{2\delta_m X_0} \tag{97}$$

In right triangle OED, $OE = OD\cos(\phi_m/2)$ and $DE = OD\sin(\phi_m/2)$.

$$|\mathbf{Y}_h| = |\mathbf{Y}_{hf}| \cos\frac{\phi_m}{2} = \frac{\cos^2(\phi_m/2)}{2\delta_m X_0} = \frac{\cos\phi_m + 1}{4\delta_m X_0} \tag{98}$$

and

$$|\mathbf{Y}_{hn}| = |\mathbf{Y}_{hf}| \sin\frac{\phi_m}{2} = \frac{\cos(\phi_m/2)\sin(\phi_m/2)}{2\delta_m X_0} = \frac{\sin\phi_m}{4\delta_m X_0} \tag{99}$$

$$\mathbf{Y}_{hf} = |\mathbf{Y}_{hf}| \,\underline{/-90^\circ + \phi_m/2} \tag{100}$$

$$\mathbf{Y}_h = |\mathbf{Y}_h| \,\underline{/-90^\circ + \phi_m} \tag{101}$$

$$\mathbf{Y}_{hn} = |\mathbf{Y}_{hn}| \,\underline{/+\phi_m} \tag{102}$$

The value of Q corresponding to the chosen $\mathbf{Y}_{hf}$ is found from Figure 31*a*:

$$\tan\phi_f = \frac{X_f}{R_f} = \frac{2\delta_m X_0}{X_0/Q} = 2\delta_m Q \tag{103}$$

and from Figure 31*b*,

$$\tan\phi_f = \cot(\phi_m/2)$$

Equating the last terms of the two equations for $\tan\phi_f$, we find the optimum value of Q to be

$$Q_o = \frac{\cot(\phi_m/2)}{2\delta_m} = \frac{\cos\phi_m + 1}{2\delta_m \sin\phi_m} \tag{104}$$

The corresponding harmonic voltage is

$$V_h = \frac{I_{hc}}{|\mathbf{Y}_h|} = \frac{4\delta_m X_0 I_{hc}}{\cos\phi_m + 1} \tag{105}$$

Table 9 shows the effect of limiting the impedance angle of the network to

Table 9

ϕ_m	0	15°	30°	45°	60°	75°	80°	85°	90°
$\delta_m Q_o$	∞	3.80	1.87	1.21	0.87	0.65	0.60	0.55	0.50
$V_h/\delta_m X_0 I_{hc}$	2.00	2.03	2.14	2.35	2.67	3.17	3.41	3.68	4.00

$\pm\phi_m$ on the optimum Q of the filter and on the maximum guaranteed harmonic voltage V_h. In particular, limitation to $\pm 75°$ reduces the harmonic voltage for a given size of filter, or the size of filter for a given harmonic voltage by about 21% from Case 2 (purely reactive network, $\phi_m = 90°$).

Typical values of Q in practice range from 30 to 60 with series resistors.

Double-tuned Filters. One double-tuned filter (the circuit of which is shown in Figure 33*b*) is substantially equivalent, near the resonant frequencies,

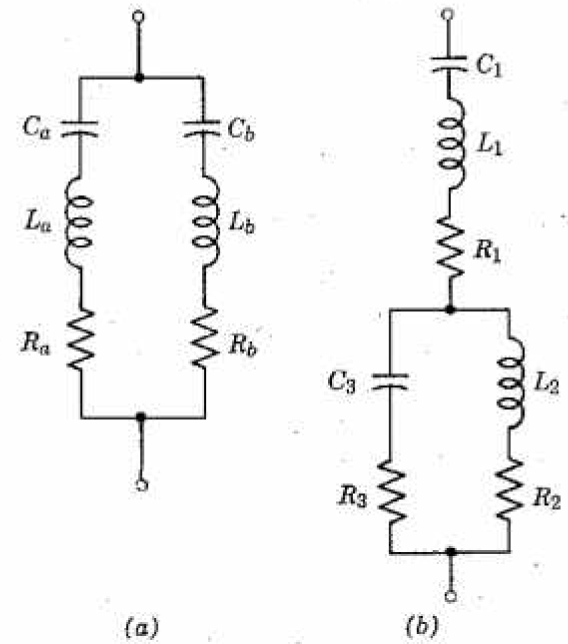


Fig. 33. Transformation from (*a*) two single-tuned filters to (*b*) double-tuned filter.

to two single-tuned filters in parallel (Figure 33*a*); for example, those tuned to the fifth and seventh harmonics. The equations for the parameters of the double-tuned filter are given by Ainsworth.[41]

The advantages of a double-tuned filter over two single-tuned filters are the following:

1. Its power loss at fundamental frequency is less.
2. One inductor, instead of two, is subjected to full impulse voltage.

Such filters are used at both terminals of the Cross Channel link.[42] The impedance of that at the French terminal is given in Figure 34*a* and *b*. Its parameters are

$R_1 = 4.2\ \Omega$	$C_1 = 1.51\ \mu F$
$R_2 = 1.656\ \Omega$	$C_3 = 12.08\ \mu F$
$R_3 = 2.11\ \Omega$	$L_1 = 208$ mH
	$L_2 = 24$ mH

Filter Components and Their Ratings

Capacitors. Capacitors account for the major part of the cost of filters. They are composed of standard units—typically rated at 100 or 150 kvar, 8 to 14.4 kV, at 50 or 60 Hz—connected in series and parallel for obtaining the desired overall voltage and kvar ratings.

Each unit consists of several rolls made of alternate layers of aluminum foil and sheets of insulation, tightly enclosed in a sheet-steel box filled with an insulating liquid. The solid insulation consists of either (a) several sheets of special paper impregnated with the liquid or (b) a sandwich of one sheet of such paper placed between two sheets of thermoplastic material. There is also an unavoidable thin film of liquid between the solid insulation and the metal foil; the thickness of the film depends on the pressure with which the rolls are formed. Three kinds of liquid impregnants are in use: (a) mineral oil, (b) trichlordiphenyl, and (c) pentachlordiphenyl. The last two are generically called *askarels*. Thus the dielectric properties depend on those of the paper, the impregnant, and the plastic (if used) and on the amounts used of each. The density of the paper can be varied, ranging from 0.8 to 1.2 g/cm^3, and the paper may be impregnated with any one of the three liquids. Of these, trichlordiphenyl is the most used at present, having superseded pentachlordiphenyl, which has a higher freezing point, a lower dielectric strength, and a lower dielectric constant.

Two of the most important properties of the capacitors are (a) temperature coefficient of capacitance and (b) reactive power per unit of volume. The latter is usually proportional to the dielectric constant and the square of the maximum safe voltage gradient. Other important properties are (c) power loss, (d) reliability (or life), and (e) cost. Approximate values of some of these properties are listed in Table 10 for various dielectric materials. These values should be interpreted as indicative rather than exact, because they vary with the density of the paper, the thickness of the liquid film, the temperature at which they are measured, quality control of the materials, etc.

A very low temperature coefficient of capacitance is desirable for tuned filters in order to avoid detuning caused by change of capacitance with ambient temperature or with self-heating of the capacitors; but this property is unimportant for damped filters or for power-factor capacitors. Capacitors filled with mineral oil can have either positive or negative temperature coefficients, depending on paper density or film thickness, and, by proper design, can be made to have essentially zero coefficient. In the past such capacitors have been used for tuned filters almost to the exclusion of other kinds. Low temperature coefficient is obtainable also by use of high-density paper impregnated with pentachlordiphenyl. Both of these kinds, however, are substantially more bulky and expensive than those of equal rating having a dielectric of plastic and paper impregnated with trichlordiphenyl. The latter

Table 10. **Typical Properties of Capacitors for Power Systems partly based on Ref. 46**

Solid Dielectric	Paper Density (g/cm³)	Impregnant	Temperature Coefficient of Capacitance (10^{-6} per deg C)	Overall Dielectric Constant at 20°C	Dissipation Factor (%)	Relative kvar per Unit Volume
Paper	1.0	Mineral oil	+250	3.6	0.17	41
Paper	1.2	Mineral oil	+400	4.2	0.19	39
Paper	0.8	Pentachlor-diphenyl	...	...	0.20	87
Paper	1.0	Pentachlor-diphenyl	−460	5.2	0.28	59
Paper	1.2	Pentachlor-diphenyl	−50	5.4	...	...
Paper	0.8	Trichlor-diphenyl	−750	5.5	0.20	100
Paper	1.0	Trichlor-diphenyl	−500	5.5	0.28	66
Paper	1.2	Trichlor-diphenyl	−100	5.5	...	...
Plastic and paper	...	Trichlor-diphenyl	−710	3.1	0.10	150 200

is acceptable for use not only as power-factor capacitors and in high-pass filters but also in automatically tuned filters (page 372).

Capacitors obtain their high reactive power per unit volume by having low losses and operating at very high voltage stress. For this reason, prolonged operation at moderate overvoltage must be avoided to prevent thermal destruction of the dielectric; and even very brief operation at high overvoltage must be avoided to prevent destructive ionization of the dielectric.

The required reactive-power rating of a capacitor is calculated as the sum of the reactive powers at each of the frequencies to which it is subjected.

Inductors. These are built with nonmagnetic cores. The inductance usually has a fixed value. The Q at the predominant harmonic frequency may be selected for lowest cost and is usually between 50 and 150. If lower Q is desired, a series resistor is used. The cost of the inductor depends mainly on the maximum rms current and the insulation level for withstanding switching surges. The required insulation level may be greatly reduced, with attendant savings in cost, by protecting the inductor by connecting a lightning arrester of suitable rating in parallel with the inductor.

Arrangement. The most economical sequence of the components of a tuned filter, from ground to line, is *R, L, C*.

Conditions Under Which Required Ratings of Filter Components Are Determined. For preventing damage to the filter components, their ratings must be based on the most severe conditions to which they may be exposed; for example, one should assume the following:

1. Highest power-frequency alternating voltage, say, 10% above nominal or normal voltage.
2. Higher effective frequency deviation than that assumed in determining adequacy of filtering, say, $\pm 5\%$.
3. Highest harmonic current, caused by resonance of the respective filter branch with the network and the other filter branches. Logically, harmonic currents from other sources than the converter in question should be assumed; however, experience has shown that, with high-power converters, the harmonics from other sources can be neglected.

Tuning

The manufacturing tolerances of inductors and capacitors are 2 to 3%. Final tuning of the tuned filters must be done after installation. The capacitance can be varied in small enough steps by changing the number of units in parallel in the tier next to ground.

The most convenient indicator is a harmonic phase-angle meter, which measures the phase difference between the harmonic voltage across the entire filter and the harmonic current through it. Such instruments have been installed in the converter stations at Sakuma, New Zealand, and Konti-Skan.[48]

The converter current should be at least 0.8 of rated current. The fifth, sixth, and seventh-harmonic filters (also seventeenth, eighteenth, and nineteenth, if used) should be tuned while the converter is in six-pulse operation. The eleventh, twelfth, and thirteenth-harmonic filters should be tuned during 12-pulse operation.

Operation with One Tuned Branch of Filter Out of Service

Such operation may become necessary or, at least, desirable when it is necessary to repair a filter component or substitute a spare component. The possibility of resonance between the remainder of the filter and the power system at the frequency for which the disconnected branch is tuned should be assessed. If this possibility appears unlikely, the converter may be operated with this branch disconnected while the harmonic voltage of this frequency is measured to determine its acceptability. Perhaps it will be necessary to run the converter at reduced load in order to make this harmonic voltage acceptable.

Minimum-cost Tuned Filter

The cost of a filter tuned for a particular harmonic varies with the size of the filter in the manner shown in Figure 35, and is least at a particular size.

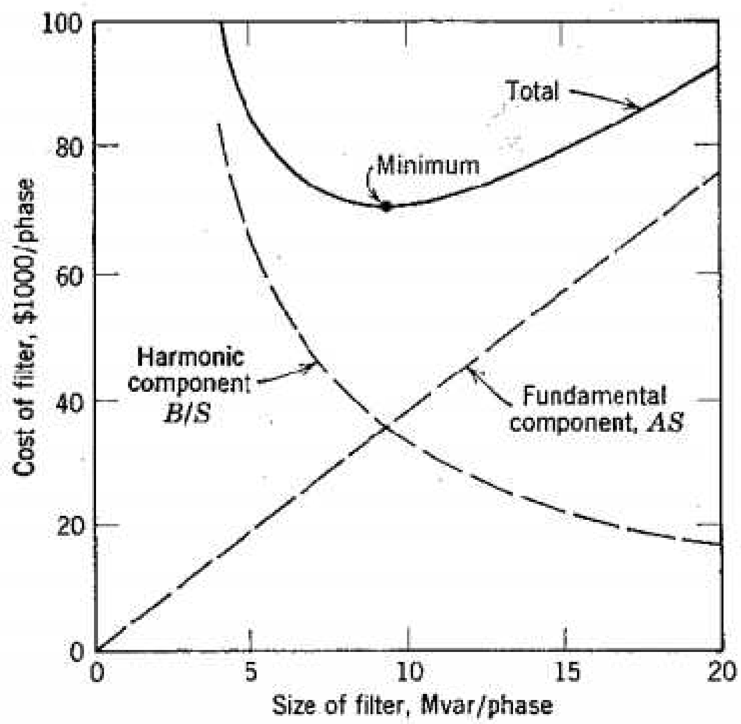


Fig. 35. Cost of filter versus its size, illustrated for fifth-harmonic filter for 600-MW 4-bridge 12-pulse converter.

The shape of the curve is attributable to the presence of two components of cost, one of which is directly proportional to size and the other, inversely proportional to size:

$$K = AS + BS^{-1} \tag{106}$$

where K = cost (k\$)
S = size (Mvar)
A, B = constants (k\$/Mvar and k\$ · Mvar, respectively)

A filter capacitor is subjected to currents and voltages of two frequencies—the fundamental power-system frequency and the harmonic frequency (of order h) for which the filter is tuned. The *rating* of the capacitor, in Mvar, must be the largest sum of the fundamental-frequency reactive power and the harmonic reactive power for which the filter is designed. Other harmonics in a tuned filter are negligible. The *cost* of the capacitor is assumed to be directly proportional to its rating. The *size* of the filter, by definition, is the reactive power of the capacitor at fundamental frequency only.

The fundamental-frequency source is essentially a constant-voltage source; the harmonic source is essentially a constant-current source. Therefore, the fundamental-frequency reactive power of the capacitor is directly proportional to its size; the harmonic reactive power is inversely proportional. The capacitor rating is

$$P_{rC} = V_1^2\omega_1 C + \frac{I_{hf}^2}{h\omega_1 C} = S + \frac{V_1^2 I_{hf}^2}{hS} \qquad \text{megavars} \qquad (107)$$

where $C =$ capacitance (F)
$\omega_1 = 2\pi \times$ fundamental frequency
$V_1 =$ fundamental voltage (kV)
$I_{hf} =$ harmonic current of order h (kA)
$S =$ size of capacitor (Mvar)

The rating of the inductor may be assumed to depend similarly on the sum of the fundamental and harmonic reactive powers:

$$P_{rL} = \frac{S}{h^2} + \frac{V_1^2 I_{hf}^2}{hS} \qquad (108)$$

The uncertainty of this assumption is considerably offset by the facts that the fundamental reactive power of the inductor is much less than its harmonic reactive power and that the total reactive power and cost of the inductor are smaller than those of the capacitor. The cost of the inductor is greatly affected by its insulation level, but this component of cost should be independent of filter size.

Neglecting the cost of the resistor, the total cost of the filter is

$$K = P_{rC}U_C + P_{rL}U_L \qquad (109)$$

where U_C and U_L are the unit costs of capacitor and inductor, respectively. Substitution of the values of P_{rC} and P_{rL} from Eqs. (107) and (108) gives

$$K = S\left(U_C + \frac{U_L}{h^2}\right) + \frac{V_1^2 I_{hf}^2}{hS}(U_C + U_L) = AS + BS^{-1} \qquad (110)$$

The size for minimum cost is found by equating the derivative dK/dS to zero:

$$\frac{dK}{dS} = A - BS^{-2} = 0 \tag{111}$$

whence

$$S_{\min} = \left(\frac{B}{A}\right)^{1/2} \tag{112}$$

and substitution of this value of S into the equation for cost gives the minimum cost as

$$K_{\min} = 2\sqrt{AB} \tag{113}$$

EXAMPLE 1

Find the minimum-cost fifth-harmonic filter for a bipolar four-bridge 12-pulse converter rated 1.00 kA, ± 300 kV on the dc side. The filters are to be connected to the 235-kV 60-Hz three-phase line. The fifth-harmonic filter is to be designed for the operation of the converter with one bridge out of service. Assume the unit cost of capacitors to be \$3.50/kvar and that of inductors \$8.00/kvar. At full load with all bridges in service, $\alpha = 15°$, $u = 25°$, and $\cos\phi = 0.866$. The limiting network impedance angle may be taken as 75°.

SOLUTION

The rated power of the converter is

$$P_n = V_{dn} I_{dn} = 600 \times 1.00 = 600 \text{ MW}$$

The full-load fundamental alternating line current is

$$I_{L1} = \frac{P_{dn}}{3V_1 \cos\phi} = \frac{600}{3(235/\sqrt{3})0.866} = 1.70 \text{ kA} = 1700 \text{ A}$$

The fifth-harmonic current from one bridge of the converter is approximately $1700/(5 \times 4) = 85.0$ A, but a more accurate value, allowing for ignition delay and overlap, is $0.165 \times 1700/4 = 70.2$ A. From Eq. (98), we find that the harmonic current in the filter is larger than that from the converter by a factor $|\mathbf{Y}_{hf}|/|\mathbf{Y}_h| = \sec(\phi_m/2) = \sec 37.5° = 1.26$; it is, therefore $1.26 \times 70.2 = 88.5$ A.

$$A = U_C + \frac{U_L}{h^2} = 3.50 + \frac{8.00}{(5)^2} = 3.82 \text{ \$/kvar}$$

$$= 3820 \text{ \$/Mvar}$$

$$B = \frac{V_1^2 I_{hf}^2 (U_C + U_L)}{h} = \frac{(235/\sqrt{3})^2 (88.5 \times 10^{-3})^2\ 11{,}500}{5}$$

$$= 3.32 \times 10^5\ \$\cdot\text{Mvar}$$

$$S_{\min} = \left(\frac{B}{A}\right)^{1/2} = \left(\frac{3.32 \times 10^5}{3.82 \times 10^3}\right)^{1/2} = 9.32 \text{ Mvar per phase}$$

is the size for minimum cost.

The corresponding cost is

$$K_{\min} = 2\sqrt{AB} = 2\sqrt{3.82 \times 10^3 \times 3.32 \times 10^5}$$

$$= \$71{,}200 \text{ per phase}$$

For any size S the cost is

$$K = \$3820S + \frac{\$332{,}000}{S}$$

The result is plotted in Figure 35.

Capacitance of the minimum-cost filter is

$$C = \frac{S_{\min}}{\omega_1 V_1^2} = \frac{9.32}{377 \times (235/\sqrt{3})^2} = 1.34 \times 10^{-6}\text{ F} = 1.34\ \mu\text{F}$$

Its inductance is

$$L = \frac{1}{C(h\omega_1)^2} = \frac{10^6}{1.34(5 \times 377)^2} = 0.210 \text{ H}$$

Optimum Q for $\phi_m = 75°$ is $0.65/\delta_m$. If δ_m is taken as 0.02, $Q_o = 0.65/0.02 = 32.5$, and the resistance of the filter is

$$R = \frac{X_0}{Q_o} = \frac{1}{Q_o}\left(\frac{L}{C}\right)^{1/2} = \frac{1}{32.5}\left(\frac{0.210}{1.34 \times 10^{-6}}\right)^{1/2} = \frac{395}{32.5} = 12.2\ \Omega$$

The fifth harmonic voltage is (by Table 9)

$$V_5 = 3.17\delta_m X_0 I_5 = 3.17 \times 0.02 \times 395 \times 88.5 = 2220 \text{ V}$$

$$= 2.22 \text{ kV}$$

This is 1.64% of the fundamental line-to-ground voltage V_1 and is greater than the usually acceptable value of 1%. For decreasing it to 1%, the size of the filter would have to be 1.64 times as great as that calculated, or 15.3 Mvar per phase. The increase of cost (Figure 35) would be moderate, \$10,000/\$71,200 = 14%.

From the curve in Figure 35 it is seen, on the one hand, that the cost of the filter increases sharply with a decrease of size below that for minimum cost and, on the other hand, that the cost increases more slowly with an increase of size above that for minimum cost. Moreover, the quality of filtering increases with size. Thus, it is reasonable to assume that a filter smaller than that for minimum cost is almost never used and that ones greater than that for minimum cost are often used.

The foregoing example points to one reason why six-pulse operation of a large converter is undesirable. The fifth-harmonic filter for full six-pulse operation would be four times as great as that designed for one bridge of four out of service. Its three-phase size would be $3 \times 4 \times 15.3 = 184$ Mvar. Since the full-load reactive power consumed by the converter is $P \tan \phi = 600 \times 0.577 = 346$ Mvar, the fifth harmonic filter would supply $185/346 = 53\%$ of the reactive power required. The seventh-harmonic filter, designed on the same basis, would supply 27%; and the two filters together would supply 80% of the full-load reactive requirement. The whole bank of filters would supply too much reactive power, especially at light load.

Automatically Tuned Filters[43]

It was shown earlier ("Design of Tuned Filters," page 355) that for each tuned filter branch there is an optimum value of Q, depending on the assumed values of maximum frequency deviation δ_m and maximum network impedance angle ϕ_m. Of these two variables, δ_m is the one that has the greater effect on Q, for Q varies inversely as δ_m. Thus high δ_m requires low Q, which increases the continual power-frequency losses and which either impairs the filtering by increasing the harmonic voltage or requires a greater capacitance and, consequently, greater cost for maintaining the same quality of filtering. Only partially offsetting this increase of cost is the fact that low Q decreases the harmonic current at resonance and thus decreases the reactive-power rating of a of a given capacitance.

One can get cheaper or better filtering by limiting the equivalent maximum frequency deviation. A large part of this equivalent deviation is caused by variation of capacitance with temperature. This part of the deviation can be limited by using capacitors with low temperature coefficient of capacitance, but this feature increases the cost of the capacitors.

Two methods have been proposed for limiting the equivalent frequency deviation.

One of these maintains the average temperature of the capacitors nearly constant by cooling them with air currents from a fan controlled thermostatically or by capacitance measurement. The other method varies either the inductance or capacitance by small steps so as to maintain the frequency deviation at small values—ideally at zero.

The capacitance can be varied by switching a variable number of capacitor units in parallel in the tier nearest to ground potential. The inductance can be varied by the use of a tapped coil and tap-changing mechanism or by a variometer (a fixed coil in series with a movable coil so that the coupling between the two coils is variable). A range of $\pm 5\%$ is usually adequate.

One proposed method of control measures the harmonic-frequency reactive power of the entire branch and decreases L (or C) whenever this exceeds a preset value or increases L (or C) whenever it is less than another preset value. In other words, there is an on-off servomechanism with a dead zone.[43]

The advantages of the automatically tuned filter over a fixed filter are that, for equally good filtering:

1. A capacitor of lower rating may be used.
2. The capacitor may be of a kind that has a high temperature coefficient of capacitance but also has a high reactive-power rating per unit of volume and per unit of cost.
3. Since it has a higher Q, the power loss is smaller.

Advantages 1 and 2 reduce the cost of the capacitor, which is the most expensive component of the filter. Advantage 3 reduces the cost of the resistor and the cost of the system losses. These cost savings are offset partially by the cost of the tuning control.

In some cases, advantage 1 cannot be realized because the filter must supply a large amount of reactive power at fundamental frequency. In such cases, however, advantages 2 and 3 are still realizable, and, in addition, the quality of filtering is improved.

Filter design follows the procedure already outlined for fixed tuned filters except that a smaller δ_m is specified, which depends on the accuracy of the automatic tuning.

Figure 36 compares the reactive-power rating of the capacitor of an automatically tuned filter for $\delta_m = 0.01$ to that of a fixed-tuned filter having the same filtering performance, that is, the same harmonic voltage at maximum frequency deviation.

Example 2

Calculate the rating of the capacitor required for the fifth-harmonic filter bank of Example 1 (a) if the filter has fixed tuning and if the maximum estimated detuning varies from 1 to 5%; and (b) if the filter is automatically tuned so as to limit the detuning to 1%. In both cases the fifth-harmonic voltage is to be limited to 1% of the fundamental voltage, and the maximum angle of the network impedance at 300 Hz is assumed to be 85°.

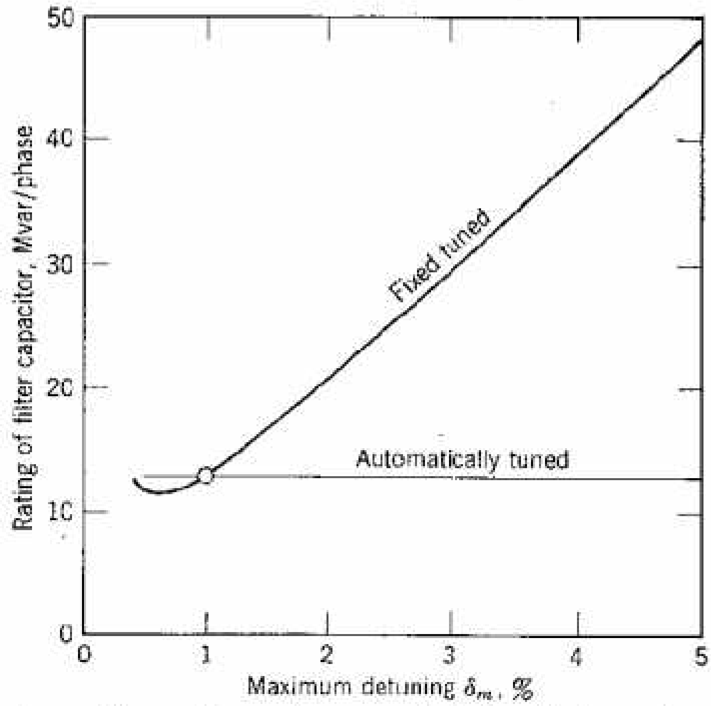


Fig. 36. Comparison of capacitor ratings required for fixed-tuned and automatically tuned filters.

SOLUTION, Part a

From Example 1, the fundamental line-to-ground voltage on the filters is $V_1 = 235/\sqrt{3} = 135.5$ kV. The fifth harmonic voltage should not exceed 1% of this, or $V_5 = 1355$ V. The fifth-harmonic current put out by the converter is 70.2 A, and the fifth-harmonic current in the filter is $I_5 = 70.2 \sec(\phi_m/2) = 70.2 \sec 42.5° = 70.2/0.737 = 95.5$ A. By Eq. (105),

$$X_0 = \frac{V_5(\cos\phi_m + 1)}{4\delta_m I_5} = \frac{1355 \times 1.087}{4\delta_m \times 95.5} = \frac{3.86}{\delta_m} \qquad \text{ohms}$$

The capacitance of the capacitor of the filter is

$$C = \frac{1}{\omega_5 X_0} = \frac{1}{5\omega_1 X_0}$$

The fundamental-frequency reactive power ("size") of the filter is

$$P_{rC1} = V_1^2\omega_1 C = \frac{V_1^2\omega_1}{5\omega_1 X_0} = \frac{V_1^2}{5X_0} = \frac{V_1^2\delta_m}{5 \times 3.86} = \frac{(135.5)^2\delta_m}{19.3}$$

$$= 950\delta_m \text{ Mvar}$$

The harmonic-frequency reactive power is

$$P_{rC5} = I_5^2 X_0 = (95.5)^2 \times \frac{3.86}{\delta_m} = \frac{3.52 \times 10^4}{\delta_m} \quad \text{vars}$$

and the capacitor rating should be

$$P_{rC} = P_{rC1} + P_{rC5} = 950\delta_m + \frac{0.0352}{\delta_m} \quad \text{Mvar}$$

The result is plotted in Figure 36.

SOLUTION, Part b

This is the same as the result of part a for $\delta_m = 0.01$.

Design of High-pass Damped Filters

Figure 37 shows three kinds. The first-order filter, a series *RC* circuit,

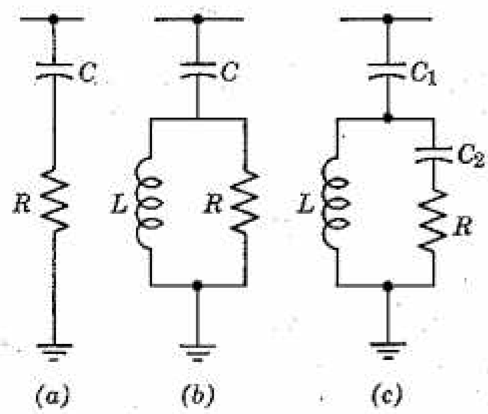


Fig. 37. High-pass damped filters: (*a*) first order, (*b*) second order, (*c*) third order.

requires a large capacitor and has excessive power loss at fundamental frequency. The second- or third-order filter is used. Both are built with low *Q*, 0.7 to 1.4, and have capacitive reactance at fundamental frequency and low, predominantly resistive, impedance over a wide band of higher frequencies. See Figures 38 to 43. The resonant frequency can be chosen near the first pair of characteristic harmonics for which tuned filters are not provided.

Impedance. The impedance of the second-order filter is

$$\mathbf{Z}_{2\text{nd}} = \frac{1}{j\omega C} + \left(\frac{1}{R} + \frac{1}{j\omega L}\right)^{-1} \tag{114}$$

and that of the third-order filter with two equal capacitors is

$$\mathbf{Z}_{3\text{rd}} = \frac{1}{j\omega C_1} + \left(\frac{1}{R + 1/j\omega C_2} + \frac{1}{j\omega L}\right)^{-1} \tag{115}$$

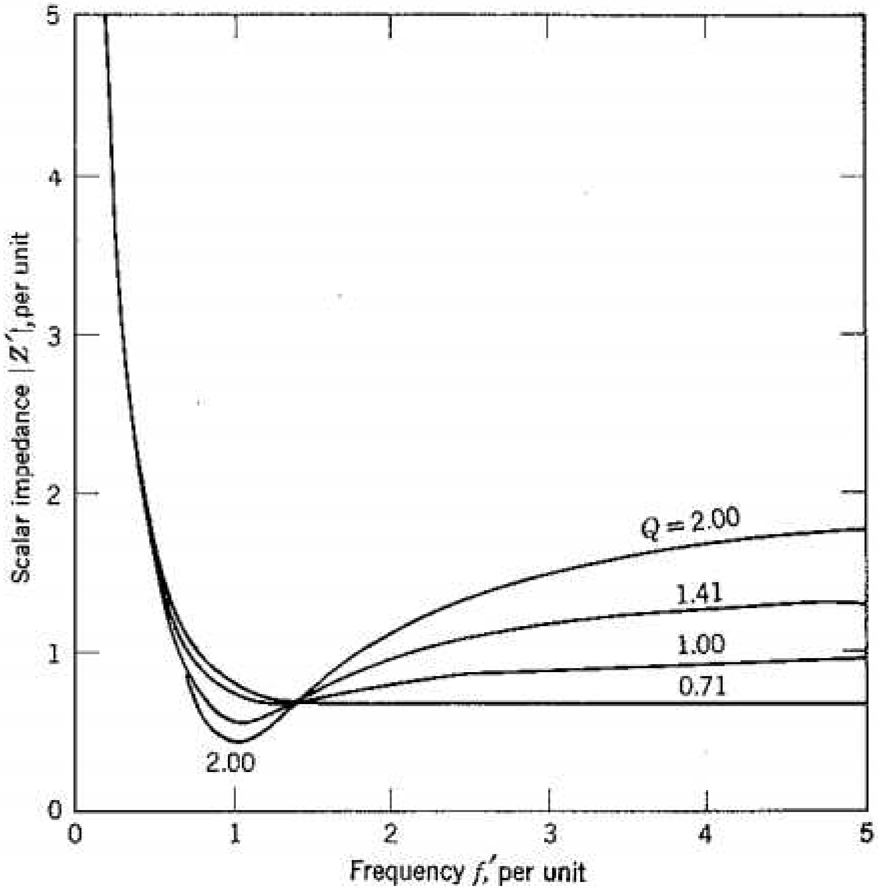


Fig. 38. Scalar impedance of second-order high-pass filters. Per-unit f' and Z' are defined by Eqs. (116) to (120).

For the sake of generality, let the following dimensionless variables be introduced:

$$\omega_n = \frac{1}{\sqrt{LC}} \tag{116}$$

$$f' = \frac{\omega}{\omega_n} = \frac{f}{f_n} \tag{117}$$

$$X_0 = \left(\frac{L}{C}\right)^{1/2} \tag{118}$$

$$Q = \frac{R}{X_0} \tag{119}$$

$$\mathbf{Z}' = \frac{\mathbf{Z}}{X_0} \tag{120}$$

These definitions are consistent with those used in connection with the tuned filter—Eqs. (73) and (75)—except that Q is defined as the reciprocal of the Q

Supply of Reactive Power by AC Filters and Shunt Capacitors

The maximum amounts of reactive power that can be accepted by and drawn from the ac network should be determined from such considerations as voltage regulation at the converter terminals and reactive-power capability of nearby generating stations. The range of this reactive power should be compared with the range of reactive power drawn by the converter between minimum and maximum loads. If the latter range does not exceed the former, it can be fitted into the former by addition of fixed shunt reactances. If the latter range exceeds the former, some of the shunt reactances must be switchable and must be switched as the load on the converter varies.

The shunt reactances must at least comprise the minimum size of filter that satisfies the requirement for satisfactory suppression of ac harmonics during the worst operating condition with respect to generation of harmonics. The filter, as already stated, presents capacitive reactance at power frequency, and thus supplies at least part of the reactive power required by the converter. The rest can be drawn from the ac network or supplied by making the filter larger or by adding shunt capacitors.

If switchable shunt reactances are required, the alternatives are the following:

1. A subdivided filter; that is, two or more duplicate filters.
2. Switchable high-pass filters in addition to those required for minimum satisfactory filtering.
3. Switchable shunt capacitors.

In theory, situations could arise in which shunt inductors would be required, but have not yet been used on existing dc links.

A larger filter than minimum is less costly than a minimum filter plus shunt capacitors and gives better filtering. If switchable elements are required, a filter plus switchable shunt capacitors is cheaper than switchable filters.

The switchable shunt reactances can be switched so as to maintain either net reactive power or alternating voltage within specified limits.

An alternative to switchable shunt reactance is a *synchronous condenser*. A condenser is more expensive than shunt capacitors but has several advantages:

1. It can absorb reactive power as well as supply it.
2. It has smooth control of reactive power or voltage instead of a few large steps.
3. It provides more stability of the alternating voltage.

Synchronous condensers have been installed at several of the existing dc links at the terminal that has no considerable nearby generating plant. They are usually connected to low-voltage tertiary windings of the converter transformers.